

Formale Systeme Proseminar

Tasks for Week 12, 7.1.2020

Task 1 Which of the following relations between $A = \{a, b, c\}$ and $B = \{1, 2\}$ are graphs of functions from A to B ?

- (a) $R_1 = \{(a, 1), (b, 2)\}$.
- (b) $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$.
- (c) $R_3 = \{(a, 1), (b, 2), (a, 2)\}$.
- (d) $R_4 = \{(a, 1), (b, 2), (c, 1)\}$.

Why?

Task 2 Let $X = \{1, 2, 3, 4, 5\}$ and consider the function $c: \mathcal{P}(X) \setminus \{\emptyset\} \rightarrow X$ defined by $c(Y) = |Y|$ for any $Y \subseteq X$, $Y \neq \emptyset$. Show that c is surjective but not injective.

Task 3 Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n + 5$ is an injection.

Task 4 Let X be any set. Show that the identity function $\text{id}_X: X \rightarrow X$ defined by $\text{id}_X(x) = x$ is a bijection.

Task 5 Let $f: A \rightarrow B$ be an injective function and $A' \subseteq A$. Prove that then

$$f^{-1}(f(A')) = A'$$

that is, prove Proposition 1 from the lecture notes on functions.

Task 6 Prove that for any set X , there exists a bijection $f: \mathcal{P}(X) \rightarrow \{0, 1\}^X$, where $\{0, 1\}^X$ is the set of all functions from X to $\{0, 1\}$.

Task 7 Prove that $f: A \rightarrow B$ is surjective if and only if it is right-cancelative: given any two functions $g: B \rightarrow C$ and $h: B \rightarrow C$ if $g \circ f = h \circ f$, then $g = h$.

The last 2-3 tasks for this week are theoretical and more difficult. Please do not worry if you can not solve them, but do try! Wish you Happy Holidays !