

# Formale Systeme Proseminar

Tasks for Week 13, 9.1.2020

**Task 1** Show that for cardinals  $|A|, |B|$  we have (as expected from the notation):  $|A| \leq |B|$  iff  $|B| \geq |A|$ .

**Task 2** Show that the function  $f: \mathbb{Z} \rightarrow \mathbb{N}$  given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection. Hence, this, together with Task 1, proves that  $|\mathbb{N}| \leq |\mathbb{Z}|$ .

**Task 3** Prove that  $A \subseteq B \Rightarrow |A| \leq |B|$ .

**Task 4** Prove by induction that

$$\forall n \in \mathbb{N} \setminus \{0, 1\}. (1 + 3 + \dots + (2n - 1) = n^2).$$

**Task 5** The sequence  $(a_i \mid i \in \mathbb{N})$  is inductively defined by

$$a_0 = 0$$

$$a_{i+1} = a_i + 3$$

Prove (by induction) that  $\forall n \in \mathbb{N}. 3 \mid a_n$ . Try to find a closed formula for  $a_n$  and prove by induction that it is really true.

**Task 6** Prove by induction that if  $A$  is a finite set, i.e.,  $|A| = k$  for some  $k \in \mathbb{N}$  then

$$|\mathcal{P}(A)| = 2^k.$$