

Formale Systeme Proseminar

Tasks for Week 13, 17.1.2019

Task 1 Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n + 5$ is an injection.

Task 2 Show that the function $f: \mathbb{Z} \rightarrow \mathbb{N}$ given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection.

Task 3 Let X be any set. Show that the identity function $\text{id}_X: X \rightarrow X$ defined by $\text{id}_X(x) = x$ is a bijection.

Task 4 Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two surjective functions. Prove that then $g \circ f$ is surjective as well.

Task 5 Prove by induction that

$$\forall n \in \mathbb{N} \setminus \{0, 1\}. (1 + 3 + \dots + (2n - 1) = n^2).$$

Task 6 Prove by induction that if A is a finite set, i.e., $|A| = k$ for some $k \in \mathbb{N}$ then

$$|\mathcal{P}(A)| = 2^k.$$