

Formale Systeme Proseminar

Tasks for Week 12, 10.1.2019

Task 1 Let $A = \{a, b, c, d\}$. For each of the following partitions of A write down the corresponding equivalence:

- (a) $\{\{a, b\}, \{c, d\}\}$,
- (b) $\{\{a\}, \{b, c, d\}\}$,
- (c) $\{\{a\}, \{b\}, \{c\}, \{d\}\}$.

Task 2 Let $A = \{a, b, c\}$. How many equivalence relations are there on A ? List them all.

Task 3 Give an example of an equivalence on \mathbb{N} with

- (a) 3 equivalence classes,
- (b) 10 equivalence classes,
- (c) 100 equivalence classes.

Task 4 Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n + 1) \mid n \in \mathbb{N}\}.$$

- (a) Find the relation R^2 ,
- (b) Find the relation R^3 ,
- (c) Can you think of a concise way to describe the reflexive and transitive closure relation R^* ?

Task 5 Which of the following relations between $A = \{a, b, c\}$ and $B = \{1, 2\}$ are graphs of functions from A to B ?

- (a) $R_1 = \{(a, 1), (b, 2)\}$.
- (b) $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$.
- (c) $R_3 = \{(a, 1), (b, 2), (a, 2)\}$.
- (d) $R_4 = \{(a, 1), (b, 2), (c, 1)\}$.

Why?

Task 6 Let $X = \{1, 2, 3, 4, 5\}$ and consider the function $c: \mathcal{P}(X) \setminus \{\emptyset\} \rightarrow X$ defined by $c(Y) = |Y|$ for any $Y \subseteq X$, $Y \neq \emptyset$. Show that

- (a) $\forall x \in X. \exists U \in \mathcal{P}(X) \setminus \{\emptyset\}. c(U) = x$,
- (b) $\neg(\forall U_1, U_2 \in \mathcal{P}(X) \setminus \{\emptyset\}. c(U_1) = c(U_2) \Rightarrow U_1 = U_2)$.