

Formale Systeme Proseminar

Tasks for Week 14, 18.1.2018

Task 1 Prove that $\aleph_0 \cdot 2 = \aleph_0$, i.e., prove that $\mathbb{N} \times \{0, 1\}$ is a countable set (a set with cardinality equal to the cardinality of \mathbb{N}).

Task 2 Prove that the following language L is at least countable, i.e., that $|L| \geq \aleph_0$ where

$$L = \{w \in \{0, 1\}^* \mid w \text{ begins with a 1 and ends with a 0}\}.$$

Task 3 Prove that the following language L is at least countable, i.e., that $|L| \geq \aleph_0$ where

$$L = \{w \in \{0, 1\}^* \mid \text{the number of 1's in } w \text{ equals two}\}.$$

Task 4 Construct a DFA for the language

$$L = \{w \in \{0, 1\}^* \mid w \text{ begins with a 1 and ends with a 0}\}.$$

Task 5 Construct a DFA for the language

$$L = \{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is preceded and followed by a } b\}.$$

Task 6 Construct a DFA for the language

$$L = \{w \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* \mid w \text{ as a natural number is divisible by 3}\}.$$

Hint: A natural number is divisible by 3 iff the sum of its digits is divisible by 3.

Task 7 Construct a DFA for the language

$$L = \{w \in \{0, 1\}^* \mid w \text{ ends with 11 or with 101}\}.$$