

Formale Systeme Proseminar

Tasks for Week 11, 14.12.2017

Task 1 Let $A = \{1, 2, 3, 4\}$ and consider the relation

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 4), (4, 3)\}.$$

- (a) Show that R is an equivalence relation.
- (b) What are the equivalence classes of R ?

Task 2 Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ given as

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid (xy > 0) \text{ or } x = y = 0\}.$$

Prove that R is an equivalence and write down its equivalence classes.

Task 3 Prove that for any set X , the diagonal relation $\Delta_X = \{(x, x) \mid x \in X\}$ is an equivalence.

Task 4 For each of the following relations on \mathbb{N} find out if it is a partial order, a strict order, a preorder, a total order, or an equivalence:

- (a) xRy if and only if $|x - y|$ is a multiple of 3.
- (b) xRy if and only if $x < 10$ and y is even.

Task 6 Let X be a set. Consider the relation R on $\mathcal{P}(X)$ defined by

$$(A, B) \in R \text{ iff } A \cap B = \emptyset.$$

Check if R is a partial order and/or an equivalence.

Task 7 Let $A = \{a, b, c\}$. How many equivalence relations are there on A ? List them all.