Formale Systeme Proseminar

Tasks for Week 7, 17.11.2016

Task 1 Rewrite each of the following formulas with a calculation to a simpler formula:

(a) $\forall x[P : T]$,
(b) $\forall x[P : F]$
(c) $\exists x[P : T]$
(d) $\exists x[P : F]$

Task 2 Rewrite each of the following formulas with a calculation to a simpler formula:

(a) $\forall x[P \vee Q : \neg P]$,
(b) $\forall x[P \wedge Q : \neg P]$,
(c) $\exists x[P \vee Q : \neg P]$,
(d) $\exists x[P \wedge Q : \neg P]$

Task 3 Show with a calculation that

(a) $\exists x[P : Q] \stackrel{val}{=} \neg\forall x[Q : \neg P]$,
(b) $\forall x[P : Q \vee R] \stackrel{val}{=} \forall x[P \wedge \neg Q : R]$.

Task 4 Is the following statement true? If yes, prove it with a calculation; if not, give a counter example.

$$\neg\exists_x[P : Q] \stackrel{val}{=} \forall_x[Q : P]$$

Task 5 Prove with a calculation that the following formula is a tautology.

$$\forall_x[P : Q \Rightarrow R] \Rightarrow (\forall_x[P : Q] \Rightarrow \forall_x[P : R])$$

Task 6 Show with derivations that the following formula is a tautology

$$((P \Rightarrow Q) \Rightarrow P) \Rightarrow ((P \Rightarrow Q) \Rightarrow Q)$$

Task 7 For each of the line numbers of your solution to Task 6, say where the proposition which occurs on that line is valid (i.e. allowed to be used).

Task 8 Give logical derivation of the following tautology.

$$(P \wedge (Q \Rightarrow R)) \Rightarrow ((P \Rightarrow Q) \Rightarrow (P \wedge R))$$