

Formale Systeme Proseminar

Tasks for Week 6, 10.11.2016

Task 1 Write the following statements as formulas with quantifiers. D is a subset of \mathbb{N} .

- (a) All elements of D are larger than or equal to 0.
- (b) All elements of D are larger than 5 and less than 15.
- (c) All elements of D are larger than 5 or all elements of D are smaller than 15.
- (d) Every pair of different elements of D differ by at least 2.

Task 2 Write the following statements as formulas with quantifiers.

- (a) For every natural number, there is a natural number which is greater than it by 5.
- (b) There is no natural number which is greater than all natural numbers.
- (c) There are two natural numbers the sum of whose squares is 40.
- (d) The sum of two natural numbers is greater than or equal to each of the two numbers.

Are the propositions true? Give an explanation.

Task 3 Is the following proposition true?

$$\forall x [x \in \mathbb{Z} : \exists y [y \in \mathbb{Z} : x + y = 0]] \Rightarrow \exists y [y \in \mathbb{Z} : \forall x [x \in \mathbb{Z} : x + y = 0]]$$

Explain your answer.

Task 4 Check which of the following propositions are equivalent independently of D where D is an arbitrary subset of \mathbb{R} .

- (a) $\exists x [x \in D : \forall y [y \in D : y \geq x]]$
- (b) $\exists l [l \in D : \forall k [k \in D : l \leq k]]$
- (c) $\exists k [k \in D : \forall m [m \in D : \neg(k < m)]]$
- (d) $\forall y [y \in D : \exists x [x \in D : y \leq x]]$

Task 5 Show with a counter example that the following properties hold.

(a) $\forall x [P : Q] \stackrel{val}{\neq} \forall x [Q : P]$

$$(b) \exists x[P : Q] \wedge \exists x[P : R] \stackrel{val}{\neq} \exists x[P : Q \wedge R]$$

Task 7 Are the following propositions true? Prove your answer.

$$(a) \forall n [n \in \mathbb{N} : \forall m [m \in \mathbb{R} : 3m + n > 3]]$$

$$(b) \forall n [n \in \mathbb{N} : \exists m [m \in \mathbb{R} : 3m + n > 3]]$$

$$(c) \exists n [n \in \mathbb{N} : \forall m [m \in \mathbb{R} : 3m + n > 3]]$$

$$(d) \exists n [n \in \mathbb{N} : \exists m [m \in \mathbb{R} : 3m + n > 3]]$$

Task 8 Is the following statement always true? Why?

$$\forall x[A(x) : B(x)] \Rightarrow \exists x[B(x)]$$

Task 9 Find a domain (i.e., a model) where both of the formulas below are true. The formulas are $\forall x \forall y \forall z [x = y \vee y = z \vee x = z]$ and $\exists x \exists y [x \neq y]$.