

Formale Systeme Proseminar

Tasks for Week 5, 3.11.16

Task 1 Show with calculations that for arbitrary sets A and B , we have $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Task 2 Check with a calculation whether the following abstract propositions are equivalent:

- (a) $((a \Rightarrow b) \Rightarrow \neg a)$ and $(\neg b \vee \neg a) \wedge (\neg b \vee b)$
- (b) $a \wedge b$ and $(\neg a \vee b) \Leftrightarrow a$

Task 3 Prove with a calculation that

- (a) $(A^c)^c = A$ for any set A
- (b) $A \cup (A \cap B) = A$ for any two sets A and B .

Task 4 Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

- (a) P and $\neg(P \vee Q)$
- (b) P and $\neg(P \Rightarrow Q)$

Task 5 Are the following statements valid? Why?

- (a) If $P \stackrel{val}{\models} Q$ and $Q \stackrel{val}{\models} R$ and $R \stackrel{val}{\models} S$, then $P \stackrel{val}{\models} S$.
- (b) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then $Q \stackrel{val}{\models} R$.
- (c) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then Q and R are incomparable.
- (d) If $P \stackrel{val}{\models} Q$, then $\neg Q \stackrel{val}{\models} \neg P$

Task 6 Show with a calculation:

- (a) $P \Rightarrow Q \stackrel{val}{\models} (P \wedge R) \Rightarrow (Q \wedge R)$
- (b) $\neg(P \Rightarrow \neg Q) \stackrel{val}{\models} (P \vee R) \wedge Q$

Task 7 Prove with a calculation that the following two formulas are comparable (i.e., one is stronger than the other or vice-versa)

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \quad \text{and} \quad (\neg P \Rightarrow Q) \Rightarrow R$$