

Formale Systeme Proseminar

Tasks for Week 9, 3.12.2015

Task 1 Let $M = \{a, b, c\}$. Give $M \times M$. Define (if possible) a relation R on M that is reflexive and symmetric, but not transitive.

Task 2 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is reflexive and transitive, but not symmetric.

Task 3 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is symmetric and transitive, but not reflexive.

Task 4 Check if the following relation is reflexive, symmetric, and/or transitive:

$$R_1 = \{(x, y) \mid x, y \in \mathbb{R}, x = 0 \wedge y \geq 0\}.$$

Task 5 Prove that for any set X , the diagonal relation $\Delta_X = \{(x, x) \mid x \in X\}$ is reflexive, symmetric, and transitive.

Task 6 For each of the following relations on \mathbb{N} find out if it is a partial order, a strict order, a preorder, a total order, or an equivalence:

- (a) xRy if and only if $|x - y|$ is a multiple of 3.
- (b) xRy if and only if $x < 10$ and y is even.

Task 7 Let X be a set. Prove that the relation R on $\mathcal{P}(X)$ defined by

$$(A, B) \in R \text{ if and only if } A \subseteq B$$

is a partial order. When is it a total order?

Task 8 Let X be a set. Consider the relation R on $\mathcal{P}(X)$ defined by

$$(A, B) \in R \text{ iff } A \cap B = \emptyset.$$

Check if R is a partial order and/or an equivalence.

Task 9 Is it possible that a relation R is both

- (a) symmetric and asymmetric?
- (b) symmetric and antisymmetric?