

Formale Systeme Proseminar

Tasks for Week 6

Task 1 Check which of the following propositions are equivalent independently of D where D is an arbitrary subset of \mathbb{R} .

- (a) $\exists x [x \in D : \forall y [y \in D : y \geq x]]$
- (b) $\exists l [l \in D : \forall k [k \in D : l \leq k]]$
- (c) $\exists k [k \in D : \forall m [m \in D : \neg(k < m)]]$
- (d) $\forall y [y \in D : \exists x [x \in D : y \leq x]]$

Task 2 Show with a counter example that the following properties hold.

- (a) $\forall x[P : Q] \stackrel{val}{\neq} \forall x[Q : P]$
- (b) $\exists x[P : Q] \wedge \exists x[P : R] \stackrel{val}{\neq} \exists x[P : Q \wedge R]$

Task 3 Rewrite each of the following formulas with a calculation to a simpler formula:

- (a) $\forall x[P : T]$,
- (b) $\forall x[P : F]$
- (c) $\exists x[P : T]$
- (d) $\exists x[P : F]$

Task 4 Rewrite each of the following formulas with a calculation to a simpler formula:

- (a) $\forall x[P \vee Q : \neg P]$,
- (b) $\forall x[P \wedge Q : \neg P]$,
- (c) $\exists x[P \vee Q : \neg P]$,
- (d) $\exists x[P \wedge Q : \neg P]$

Task 5 Show with a calculation that

- (a) $\exists x[P : Q] \stackrel{val}{\equiv} \neg \forall x[Q : \neg P]$,
- (b) $\forall x[P : Q \vee R] \stackrel{val}{\equiv} \forall x[P \wedge \neg Q : R]$.

Task 6 Is the following statement true? If yes, prove it with a calculation; if not, give a counter example.

$$\neg\exists_x[P : Q] \stackrel{val}{=} \forall_x[Q : P]$$

Task 7 Prove with a calculation that the following formula is a tautology.

$$\forall_x[P : Q \Rightarrow R] \Rightarrow (\forall_x[P : Q] \Rightarrow \forall_x[P : R])$$