

# Formale Systeme Proseminar

## Tasks for Week 5

**Task 1** Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

- (a)  $P$  and  $\neg(P \vee Q)$
- (b)  $P$  and  $\neg(P \Rightarrow Q)$

**Task 2** Are the following statements valid? Why?

- (a) If  $P \stackrel{val}{\models} Q$  and  $Q \stackrel{val}{\models} R$  and  $R \stackrel{val}{\models} S$ , then  $P \stackrel{val}{\models} S$ .
- (b) If  $P \stackrel{val}{\models} Q$  and  $P \stackrel{val}{\models} R$ , then  $Q \stackrel{val}{\equiv} R$ .
- (c) If  $P \stackrel{val}{\models} Q$  and  $P \stackrel{val}{\models} R$ , then  $Q$  and  $R$  are incomparable.

**Task 3** Show with a calculation:

- (a)  $P \Rightarrow Q \stackrel{val}{\models} (P \wedge R) \Rightarrow (Q \wedge R)$
- (b)  $\neg(P \Rightarrow \neg Q) \stackrel{val}{\models} (P \vee R) \wedge Q$

**Task 4** Prove with a calculation that the following two formulas are comparable (i.e., one is stronger than the other or vice-versa)

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \quad \text{and} \quad (\neg P \Rightarrow Q) \Rightarrow R$$

**Task 5** Write the following statements as formulas with quantifiers.  $D$  is a subset of  $\mathbb{N}$ .

- (a) All elements of  $D$  are larger than or equal to 0.
- (b) All elements of  $D$  are larger than 5 and less than 15.
- (c) All elements of  $D$  are larger than 5 or all elements of  $D$  are smaller than 15.
- (d) Every pair of different elements of  $D$  differ by at least 2.

**Task 6** Write the following statements as formulas with quantifiers.

- (a) For every natural number, there is a natural number which is greater than it by 5.

- (b) There is no natural number which is greater than all natural numbers.
- (c) There are two natural numbers the sum of whose squares is 40.
- (d) The sum of two natural numbers is greater than or equal to each of the two numbers.

Are the propositions true? Give an explanation.

**Task 7** Is the following proposition true?

$$\forall x [x \in \mathbb{Z} : \exists y [y \in \mathbb{Z} : x + y = 0]] \Rightarrow \exists y [y \in \mathbb{Z} : \forall x [x \in \mathbb{Z} : x + y = 0]]$$

Explain your answer.