Formale Systeme Proseminar

Tasks for Week 2 - October 15, 2015

Task 1 We are given 3 piles of chocolate. The first consists of 4 bars, the second of 6 bars, and the third of 14 bars. The piles should be evened, so that each pile consists of 8 bars. In each step one may only move chocolate bars from one pile to another. In addition, in one step one may only move n bars from pile x to pile y, if before the move bar y contained exactly n bars. Model the problem as in the example considered in class.

Task 2 Model a simple coffee&tea vending machine with three buttons (for choosing coffee, tea, or canceling an operation) and a socket for inserting coins. You may assume that there exists a single admissible coin (e.g. 1 EUR) and every drink costs the same. Hence, no money exchange happens. Describe the relevant objects being modeled and the choices made in your design of the machine.

Task 3 Consider the following sets:

$A = \{a, b, c, d, e, f\}$,

$B = \{a, c, e, f\}$,

$C = \{b, d, g, h\}$,

$D = \{c, a, f, e\}$.

1. Construct the intersection of any two of the given sets.
2. Construct the union of any two of the given sets.
3. Which sets are disjoint, which are subsets of another set, which are proper subsets of another set?

Task 4 Consider the set $S_{\mathbb{N}} = \mathcal{P}(\mathbb{N})$.

1. Write down 3 elements of $S_{\mathbb{N}}$ that are finite sets.
2. Write down an element of $S_{\mathbb{N}}$ that is an infinite set.
3. Find two disjoint subsets of natural numbers, i.e., elements of $S_{\mathbb{N}}$ whose union equals $\mathbb{N}$.

Task 5 Let $S = \{1, 2, 3\}$ and $T = \{0, 1\}$. Write down the following sets by listing their elements and provide their cardinality.

1. $A = \{x | x \in S$ and x $\neq 2\}$

2. $B = \mathcal{P}(T)$
3. $C = S \cap T$
4. $D = \mathcal{P}(C)$
5. $E = \mathcal{P}(D) = \mathcal{P}(\mathcal{P}(C))$
6. The set of all powers of 2 that are larger than 1 and smaller than 500.

Task 6 Prove that for any sets X and Y, we have $X \cap Y \subseteq X$.

Task 7 Prove that for any set X, we have $X \cup X = X$.

Task 8 Prove that for any set X there exist sets Y and Z such that $X = Y \cup Z$.

Task 9 Prove that $\emptyset \subseteq X$ for any set X.