Formale Systeme Proseminar

Tasks for Week 15, 28.1.2016

Task 1 Construct an NFA for the language

$$L = \{w \in \{a, b\}^* \mid w \text{ has at least three } a\text{'s or at least two } b\text{'s}\}.$$

Note that this language is a union of two languages.

Task 2 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is $\{0, 1\}$.

(a) The language $\{0\}$ with two states.

(b) The language $\{0\}^*$ with one state.

(c) The language $\{w \mid w \text{ ends with a } 00\}$ with three states,

(d) The language $\{1\}^* \cdot \{001^n \mid n > 0\}^*$ with three states.

Note that the regular expressions for these languages are: (a) 0; (b) 0^*; (c) $(0 \cup 1)^*00$; and (d) $1^* \cdot (001^+)^*$.

Task 3 Construct an NFA for the language

$$L = \{w_1 w_2 \in \{0, 1\}^* \mid w_1 = 0^{2n}, w_2 = 0^{3m}, \text{ for some } n, m \in \mathbb{N}\}.$$

Note that the regular expression for L is $(00)^* \cdot (000)^*$.

Task 4 Construct an NFA for the language L^* where

$$L = \{01\} \cup \{(00)^n 11 \mid n \in \mathbb{N}\}.$$

Note that the regular expression for L is $01 \cup (00)^*11$.

Task 5 Determinize the automaton from Task 4.

Task 6 Construct an NFA for the language $L_1 \cdot L_2$ where $L_1 = \{a, b\}^*$ and $L_2 = \{aabab\}$.

Task 7 Construct a DFA for the language from Task 6.

Task 8 Let L be a regular language, $L \subseteq \Sigma^*$. Show that the reversed language of L defined as

$$L^R = \{w \in \Sigma^* \mid w^R \in L\}$$

where reversed words are defined inductively by

$$\varepsilon^R = \varepsilon, (ua)^R = au^R \text{ for } a \in \Sigma, u \in \Sigma^*$$

is regular as well.

Hint: From an automaton for L, construct an automaton for L^R.

Task 9 Let $\Sigma = \{0,1\}$ and let

$$D = \{w \in \{0,1\}^* \mid \#_{01}(w) = \#_{10}(w)\}.$$

Thus $101 \in D$ because 101 contains a single 10 and a single 01, but $1010 \notin D$ because $\#_{01}(1010) = 1$ but $\#_{10}(1010) = 2$.

Show that D is a regular language.