Formale Systeme Proseminar

Tasks for Week 12, 7.1.2016

Task 1 Show that the function $f\colon \mathbb{N} \to \mathbb{N}$ given by $f(n) = n + 3$ is an injection.

Task 2 Show that the function $f\colon \mathbb{Z} \to \mathbb{N}$ given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection.

Task 3 Let X be any set. Show that the identity function $\mathrm{id}_X\colon X \to X$ defined by $\mathrm{id}_X(x) = x$ is a bijection.

Task 4 Let $f\colon A \to B$ and $g\colon B \to C$ be two injective functions. Prove that then $g \circ f$ is injective as well. (Hence, you need to prove Lemma I4 from the lectures.)

Task 5 Prove that $f\colon A \to B$ is surjective if and only if it is right-cancelative: given any two functions $g\colon B \to C$ and $h\colon B \to C$ if $g \circ f = h \circ f$, then $g = h$.

Task 6 Prove by induction that

$$\forall n \in \mathbb{N} \setminus \{0, 1\}.(1 + 3 + \ldots + (2n - 1) = n^2).$$

Task 7 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$a_0 = 0$
$a_{i+1} = a_i + 3$

Prove (by induction) that $\forall n \in \mathbb{N}.3|a_n$. Try to find a closed formula for a_n and prove by induction that it is really true.

Task 8 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$a_0 = 1$
$a_{i+1} = \frac{1}{i+1} \sum_{k=0}^{i} a_k$

Prove (by induction) that $\forall n \in \mathbb{N}.a_n = 1$.

Task 9 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$a_0 = 2$
$a_{i+1} = 2a_i - 1$

Prove (by induction) that $\forall n \in \mathbb{N}.a_n = 2^n + 1$.