Formale Systeme Proseminar

Week 10, 10.12.2015

Task 1 Let $A = \{1, 2, 3, 4\}$ and consider the relation

$$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (3,4), (4,3)\}.$$

(a) Show that R is an equivalence relation.

(b) What are the equivalence classes of R?

Task 2 Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ given as

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid \ (xy > 0) \text{ or } x = y = 0\}.$$

Prove that R is an equivalence and write down the equivalence classes of R.

Task 3 Describe the equivalence classes of the equivalence $\equiv_5$ on $\mathbb{Z}$ defined in the lectures. In general, for a fixed natural number n, describe the classes of $\equiv_n$. How many classes are there?

Task 4 Show that the relation **on** $\mathbb{N} \times \mathbb{N}$ defined by

$$(a, b)R(c, d) \text{ if and only if } a + d = b + c$$

is an equivalence.

Task 5 Let $A = \{a, b, c, d\}$. For each of the following partitions of A write down the corresponding equivalence:

(a) $\{\{a, b\}, \{c, d\}\}$,

(b) $\{\{a\}, \{b, c, d\}\}$,

(c) $\{\{a\}, \{b\}, \{c\}, \{d\}\}$.

Task 6 Let $A = \{a, b, c\}$. How many equivalence relations are there on A? List them all.

Task 7 Give an example of an equivalence on $\mathbb{N}$ with

(a) 3 equivalence classes,

(b) 10 equivalence classes,

(c) 100 equivalence classes.

Task 8 Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n+1) \mid n \in \mathbb{N}\}.$$

(a) Find the relation R^2,

(b) Find the relation R^3,

(c) Can you think of a concise way to describe the reflexive and transitive closure relation R^*? (Please read the definition on the last slide of Week 9 – we will discuss it on Wednesday in Week 10.)