

Formale Systeme Proseminar

Tasks for Week 9

Task 1 Prove that:

- (a) $P \Rightarrow Q$ is not equivalent to $Q \Rightarrow P$
- (b) $P \Rightarrow Q$ is not equivalent to $\neg P \Rightarrow \neg Q$
- (c) $P \Leftrightarrow Q \Leftrightarrow R$ is not equivalent to $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

Remember this!

Task 2 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

- (a) $P \vee (\neg P \wedge Q) \stackrel{val}{=} P \vee Q$
- (b) $P \wedge (P \Rightarrow Q) \stackrel{val}{=} P \wedge Q$
- (c) $P \vee (P \wedge Q) \stackrel{val}{=} P$
- (d) $P \wedge (P \vee Q) \stackrel{val}{=} P$
- (e) $P \Rightarrow \neg Q \stackrel{val}{=} \neg(P \wedge Q)$

Task 3 Show with a calculation that the following formulas are tautologies

- (a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$
- (b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 4 Show with calculations that for arbitrary sets A and B , we have $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Task 5 Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

- (a) $P \vee Q$ and $P \wedge Q$
- (b) P and $\neg(P \vee Q)$
- (c) P and $\neg(P \Rightarrow Q)$

Task 6 Are the following statements valid? Why?

- (a) If $P \stackrel{val}{=} Q$ and $Q \stackrel{val}{=} R$ and $R \stackrel{val}{=} S$, then $P \stackrel{val}{=} S$.

- (b) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then $Q \stackrel{val}{=} R$.
- (c) If $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$, then Q and R are incomparable.