Formale Systeme Proseminar

Tasks for Week 7

Task 1 Show that the function $f\colon \mathbb{N} \to \mathbb{N}$ given by $f(n) = n + 3$ is an injection.

Task 2 Show that the function $f\colon \mathbb{Z} \to \mathbb{N}$ given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection.

Task 3 Let X be any set. Show that the identity function $\mathrm{id}_X\colon X \to X$ defined by $\mathrm{id}_X(x) = x$ is a bijection.

Task 4 Prove Proposition S3 from the lectures, that is, show that if $f\colon A \to B$ is a surjective function and $B' \subseteq B$ then $f(f^{-1}(B')) = B'$.

Task 5 Let $f\colon A \to B$ and $g\colon B \to C$ be two injective functions. Prove that then $g \circ f$ is injective as well. (Hence, you need to prove Lemma I4 from the lectures.)

Task 6 Let X and Y be finite sets with $|X| = |Y|$. Prove that every injective function $f\colon X \to Y$ must also be surjective (and hence bijective).

Task 7 Let R be an equivalence relation on a set X. Show that the assignement $c(x) = [x]_R$ defines a surjective function $c\colon X \to X/R$.

Task 8 Prove that $f\colon A \to B$ is surjective if and only if it is right-cancelative: given any two functions $g\colon B \to C$ and $h\colon B \to C$ if $g \circ f = h \circ f$, then $g = h$.

Task 9 Prove that $f\colon A \to B$ is injective if and only if it has a left inverse: there exists a function $g\colon B \to A$ with the property that $g \circ f = \mathrm{id}_A$.