

# Formale Systeme Proseminar

Week 5, 30.10.2014

**Task 1** For each of the following relations on  $\mathbb{N}$  find out if it is a partial order, a strict order, a preorder, a total order, or an equivalence:

- (a)  $xRy$  if and only if  $|x - y|$  is a multiple of 3.
- (b)  $xRy$  if and only if  $x < 10$  and  $y$  is even.

**Task 2** Let  $X$  be a set. Prove that the relation  $R$  on  $\mathcal{P}(X)$  defined by

$$(A, B) \in R \text{ if and only if } A \subseteq B$$

is a partial order. When is it a total order?

**Task 3** Prove that the relation  $\nabla_X = X \times X$  is an equivalence relation for any set  $X$ . How many classes does  $\nabla_X$  have? How many classes does  $\Delta_X$  have?

**Task 4** Let  $X$  be a set. Consider the relation  $R$  on  $\mathcal{P}(X)$  defined by

$$(A, B) \in R \text{ iff } A \cap B = \emptyset.$$

Check if  $R$  is a partial order and/or an equivalence.

**Task 5** Let  $X$  be a non-empty set. Prove that the only relation on  $X$  that is both a partial order and an equivalence is  $\Delta_X$ .

**Task 6** Let  $A = \{1, 2, 3, 4\}$  and consider the relation

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 4), (4, 3)\}.$$

- (a) Show that  $R$  is an equivalence relation.
- (b) What are the equivalence classes of  $R$ ?

**Task 7** Consider the relation  $R \subseteq \mathbb{Z} \times \mathbb{Z}$  given as

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid (xy > 0) \text{ or } x = y = 0\}.$$

Prove that  $R$  is an equivalence and write down the equivalence classes of  $R$ .

**Task 8** Show that the relation **on**  $\mathbb{N} \times \mathbb{N}$  defined by

$$(a, b)R(c, d) \text{ if and only if } a + d = b + c$$

is an equivalence.

**Task 9** Let  $A = \{a, b, c\}$ . How many equivalence relations are there on  $A$ ? List them all.