Formale Systeme Proseminar

Tasks for Week 4, 23.10.2014

Task 1 Prove the following property for any two sets A and B:

If $A \cup B = \emptyset$ then $A = B = \emptyset$.

Does it hold that for any two sets A and B, if $A \cap B = \emptyset$ then $A = B = \emptyset$? If so, prove the property; if not give a counterexample.

Task 2 Check whether the following propositions always hold. If so, give a proof; if not, give a counterexample:

1. If $A \subseteq B$, then $A \cup B = A$.
2. If $A \subseteq B$, then $A \cap B = A$.

Task 3 Check whether the following proposition always holds. If so, give a proof; if not, give a counterexample:

$\mathcal{P}(A) \times \mathcal{P}(B) = \mathcal{P}(A \times B)$.

Task 4 Prove the following property for any two sets A and B.

$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

Task 5 Let $M = \{a, b, c\}$. Give $M \times M$. Define (if possible) a relation R on M that is reflexive and symmetric, but not transitive.

Task 6 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is reflexive and transitive, but not symmetric.

Task 7 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is symmetric and transitive, but not reflexive.

Task 8 Check if each of the following relations is reflexive, symmetric, and/or transitive:

(a) $R_1 = \{(x, y) \mid x, y \in \mathbb{R},\ x = 0 \text{ and } y \geq 0\}$,

(b) $R_2 = \{(u, v) \mid u, v \in A^* \text{ and } u \text{ is a prefix of } v\}$.

Task 9 Prove that for any set X, the diagonal relation $\Delta_X = \{(x, x) \mid x \in X\}$ is reflexive, symmetric, and transitive.