

# Formale Systeme Proseminar

## Tasks for Week 15

**Task 1** Prove that the set  $2\mathbb{N} = \{2n \mid n \in \mathbb{N}\}$  is countable.

**Task 2** Prove that  $\aleph_0 \cdot 2 = \aleph_0$ , i.e., prove that  $\mathbb{N} \times \{0, 1\}$  is a countable set.

**Task 3** Prove that every class of the equivalence relation  $\equiv_3$  on  $\mathbb{Z}$  is countable.

**Task 4** Construct a DFA for the language

$$L = \{w \in \{0, 1\}^* \mid w \text{ begins with a 1 and ends with a 0}\}.$$

**Task 5** Construct a DFA for the language

$$L = \{w \in \{a, b\}^* \mid \text{every } a \text{ in } w \text{ is preceded and followed by a } b\}.$$

**Task 6** Construct a DFA for the language

$$L = \{w \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* \mid w \text{ as a natural number is divisible by 3}\}.$$

Hint: A natural number is divisible by 3 iff the sum of its digits is divisible by 3.

**Task 7** Construct a DFA for the language

$$L = \{w \in \{0, 1\}^* \mid w \text{ ends with 11 or with 101}\}.$$

**Task 8** Let  $L$  be the language of all strings over  $\{0, 1\}$  that do not contain a pair of 1's that are separated by an odd number of symbols. Give the state diagram of a DFA with 5 states that recognizes  $L$ .