

# Formale Systeme Proseminar

## Tasks for Week 13

**Task 1** Show with derivations that the following formula is a tautology

$$\exists x \forall y [P(x) \Rightarrow Q(y)] \Rightarrow (\forall u [P(u)] \Rightarrow \exists v [Q(v)])$$

**Task 2** Prove with a derivation that the following formula is a tautology.

$$\exists y [\forall x [P(x) \wedge Q(x, y)]] \Rightarrow \forall z [P(z)]$$

**Task 3** Prove with a derivation that the following formula is a tautology.

$$\exists x [\forall y [P(x, y)]] \Rightarrow \forall v [\exists u [P(u, v)]]$$

**Task 4** Show that there is unique (up to isomorphism) group on a two-element set  $\{0, 1\}$ .

**Task 5** Find all rings (up to isomorphism) on a two-element set  $\{0, 1\}$ .

**Task 6** Prove that  $\equiv_n$ , defined as usual by  $k \equiv_n m$  iff  $n \mid (m - k)$ , is a congruence on the ring of integers  $\mathbb{Z}(+, \cdot)$ .

**Task 7** Prove that the quotient algebra  $\mathbb{Z}_n(+_n, \cdot_n)$  is a ring using the isomorphism theorem.

**Task 8** Give an example of a groupoid on the set  $\{0, 1, 2\}$  that is:

- (a) commutative,
- (b) with a unit 2,
- (c) cancellative.