Formale Systeme Proseminar

Tasks for Week 10

Task 1 Show with a calculation:

(a) $P \Rightarrow Q \overset{val}{\models} (P \wedge R) \Rightarrow (Q \wedge R)$

(b) $\neg(P \Rightarrow \neg Q)) \overset{val}{\models} (P \vee R) \wedge Q$

Task 2 Prove with a calculation that the following two formulas are comparable (i.e., one is stronger than the other or vice-versa)

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \quad \text{and} \quad (\neg P \Rightarrow Q) \Rightarrow R$$

Task 3 Write the following statements as formulas with quantifiers. D is a subset of $\mathbb{N}$.

(a) All elements of D are larger than or equal to 0.

(b) All elements of D are larger than 5 and less than 15.

(c) All elements of D are larger than 5 or all elements of D are smaller than 15.

(d) Every pair of different elements of D differ by at least 2.

Task 4 Write the following statements as formulas with quantifiers.

(a) For every natural number, there is a natural number which is greater than it by 5.

(b) There is no natural number which is greater than all natural numbers.

(c) There are two natural numbers the sum of whose squares is 40.

(d) The sum of two natural numbers is greater than or equal to each of the two numbers.

Are the propositions true? Give an explanation.

Task 5 Write the following propositions as a formulas with connectives and quantifiers. You may use that $\mathbb{P}$ denotes the set of all prime numbers.

(a) Every sum of three prime numbers is also a prime number.

(b) All prime numbers are even, except the number 3.

(c) There is a prime number which is 1 plus a multiple of 6.

Are the propositions true? If yes, give an explanation; if not, give a counter example.

Task 6 Check which of the following propositions are equivalent independently of D where D is an arbitrary subset of $\mathbb{R}$.

(a) $\exists x \; [x \in D : \forall y \; [y \in D : y \geq x]]$

(b) $\exists l \; [l \in D : \forall k \; [k \in D : l \leq k]]$

(c) $\exists k \; [k \in D : \forall m \; [m \in D : \neg(k < m)]]$

(d) $\forall y \; [y \in D : \exists x \; [x \in D : y \leq x]]$

Task 7 Show with a counter example that the following properties hold.

(a) $\forall x[P : Q] \overset{val}{\neq} \forall x[Q : P]$

(b) $\exists x[P : Q] \wedge \exists x[P : R] \overset{val}{\neq} \exists x[P : Q \wedge R]$

Task 8 Is the following proposition true?

$$\forall x \; [x \in \mathbb{Z} : \exists y \; [y \in \mathbb{Z} : x + y = 0]] \Rightarrow \exists y \; [y \in \mathbb{Z} : \forall x \; [x \in \mathbb{Z} : x + y = 0]]$$

Explain your answer.