

Formale Systeme Proseminar

Tasks for Week 7

Task 1 Check in each of the following cases whether the given rule is correct. If it is, give arguments to show this. If it is not, give a counter example.

- (a) There are K's which are also M's
All K's are L's

There are L's which are M's

- (b) No one K is an M
All K's are L's

No one L is an M

Task 2 For each of the following concrete propositions, write an abstract proposition which corresponds to it:

- (a) I love you and will always be true to you.
(b) If it is raining, then I will stay home and watch a movie.
(c) $x^2 > 4$ if, and only if, $x > 2$ or $x < -2$.
(d) I will go to play tennis if you bring the balls with you.

Task 3 Give the following propositions in words again, with 'it is raining' for a , 'it is windy' for b , and 'I am wet' for c .

- (a) $a \wedge \neg b$
(b) $\neg(a \vee b)$
(c) $(a \Rightarrow c) \vee (b \Rightarrow \neg a)$.
(d) $\neg\neg a$.

Task 4 Draw the trees of the following abstract propositions and give the main symbol for each of them.

- (a) $(a \Rightarrow (b \Rightarrow a))$
(b) $((\neg(a \Rightarrow b)) \Leftrightarrow (a \wedge (\neg b)))$
(c) $((\neg(\neg a)) \Rightarrow ((\neg a) \wedge b))$

(d) $(a \Rightarrow ((b \wedge a) \vee c))$.

Task 5 Drop as many parenthesis as possible from the abstract propositions of Task 4.

Task 6 Give the truth tables of the abstract propositions of Task 4.

Task 7 For which values of a, b , and c one gets 0 in the truth-table of

$$(a \wedge (b \Rightarrow c)) \Rightarrow ((b \Rightarrow a) \wedge c) ?$$

Task 8 Show the equivalence of

- (a) $\neg(b \vee \neg c)$ and $\neg b \wedge c$
- (b) $\neg(a \Leftrightarrow b)$ and $\neg a \Leftrightarrow b$
- (c) $(a \vee b) \wedge a$ and a
- (d) $(a \vee b) \wedge b$ and $(b \wedge c) \vee (b \wedge \neg c)$.

Task 9 Give an example of a tautology with only one proposition variable a , without T or F , and with only parentheses and

- (a) connective \Rightarrow
- (b) connectives \vee and \neg
- (c) connectives \wedge and \neg
- (d) connective \Leftrightarrow .