

# Formale Systeme Proseminar

## Tasks for Week 14

**Task 1** Prove that the set  $2\mathbb{N} = \{2n \mid n \in \mathbb{N}\}$  is countable.

**Task 2** Prove that  $\aleph_0 \cdot 2 = \aleph_0$ , i.e., prove that  $\mathbb{N} \times \{0, 1\}$  is a countable set.

**Task 3** Prove that every class of the equivalence relation  $\equiv_6$  on  $\mathbb{Z}$  is countable.

**Task 4** Prove that the set of all pairwise disjoint balls in  $\mathbb{R}^2$  is countable.

Hint: You may use that  $\mathbb{Q}$  is dense in  $\mathbb{R}$ , i.e., that between any two different reals there is a rational number. You should also use that  $\mathbb{Q}$  is countable (proven in class).

**Task 5** Prove that the set  $\mathbb{Q}_{sq} = \{q^2 \mid q \in \mathbb{Q}\}$  is countable.

**Task 6** Prove that  $|\mathbb{R}^+| = |(0, 1)|$ .

**Task 7** Prove that  $\mathbb{N}^{\mathbb{N}}$  is uncountable.