Formale Systeme Proseminar

Tasks for Week 12

Task 1 Let $A = \{1, 2, 3\}$. Give an example of a binary operation $*$ on A so that $A(*)$ is:

(1) a commutative groupoid.

(2) a cancellative groupoid.

Task 2 Find all subgroupoids of the groupoid given by the following table

G(*)	a	b	c
a	b	a	c
b	b	a	c
c	a	b	a

Task 3 Give an example of a group with 3 elements. (Hint: See Task 8.)

Task 4 Let A be a set and consider the set $P(A) = \{f : A \to A \mid f \text{ is bijective }\}$. Prove that $P(A)(\circ)$ is a group, the group of permutations, with $\circ$ being the usual composition of functions.

Task 5 Show that the group of permutations $P(A)(\circ)$ is not commutative unless $|A| = 1$ or $|A| = 2$.

Task 6 Let $A = \{0, 1\}$. Prove that none of the groupoids $(\mathcal{P}(A), \cap)$, $(\mathcal{P}(A), \cup)$, $(\mathcal{P}(A), \setminus)$ is a group.

Task 7 Let $G(*)$ be a nonempty semigroup with the property

$$\forall a \in G.(\exists b \in G.(\forall x \in G.axb = x)).$$

Prove that $G(*)$ is a commutative group.

Task 8 Consider the equivalence $\equiv_n$ on $\mathbb{Z}$ defined (as usual) by

$$k \equiv_n m \text{ if and only if } n|(k - m).$$

Show that $\equiv_n$ is a congruence of the ring $(\mathbb{Z}, +, -(-), 0, \cdot, 1)$. The quotient ring $\mathbb{Z}/\cong_n$ is denoted by $\mathbb{Z}_n$. Write down the Cayley tables of $\mathbb{Z}_n$ for $n = 3$.