

# Formale Systeme

Example test tasks, to be discussed on 6.12.2019

**Task 1.** (20 points) Prove that the following propositional formula is a tautology in at least two different ways (via a truth table, or via a calculation, or via a derivation):

$$(P \wedge Q) \Rightarrow (P \Rightarrow Q)$$

**Task 2.** (20 points) Prove that for arbitrary sets  $A, B, C$ :

$$A \subseteq B \Rightarrow A \cap C \subseteq B \cap C.$$

**Task 3.** (20 points) Consider the predicates:

$$E(x) \stackrel{val}{=} 1 \leq x \leq 10 \wedge 2|x$$

and

$$S(x, 10) \stackrel{val}{=} \exists y[3 \leq y \leq 7 : x + y = 10].$$

Is the formula

$$\forall x[E(x) : S(x, 10)]$$

true? Explain your answer in detail.

**Task 4.** (20 points) Prove that the following predicate formula is a tautology:

$$\forall x[P(x) : Q(x)] \wedge \exists x[A(x) : P(x)] \Rightarrow \exists x[A(x) : Q(x)].$$

**Task 5.** (30 points) Let  $\mathbb{P}$  denote the set of all prime (natural) numbers. Write the following statements as predicate formulas:

- (a) Every prime number different than 2 is odd.
- (b) Every prime number larger than 1000 can be written as a sum of two other prime numbers.
- (c) There are infinitely many prime numbers.