

# Formale Systeme

Example Test 2, to be discussed in the Q&A Session on January 25, 2019

**Task 1.** (20 points) Define the following notions:

- A relation  $R$  is transitive.
- A relation  $R$  is an equivalence.
- A function  $f$  is surjective.
- $|A| < |B|$  for two sets  $A$  and  $B$ .

**Task 2.** (20 points) Let  $X$  and  $Y$  be nonempty sets and  $f: X \rightarrow Y$  a function. Consider the relation  $\ker f \subseteq X \times X$  defined by

$$(x_1, x_2) \in \ker f \iff f(x_1) = f(x_2).$$

Then  $\ker f$  is an equivalence relation (you need not prove this).

For  $X = Y = \{a, b, c\}$  and  $f: X \rightarrow Y$  given by  $f(a) = a$ ,  $f(b) = b$ ,  $f(c) = b$  give the equivalence classes of  $\ker f$ .

Prove that  $\ker f = \Delta_X$  if and only if  $f$  is injective.

**Task 3.** (20 points) Let  $\Sigma = \{0, 1\}$  and consider the prefix relation  $\preceq$  on  $\Sigma^*$  defined by

$$u \preceq v \iff \exists w \in \Sigma^*. v = uw.$$

Prove that  $\preceq$  is a partial order.

**Task 4.** (20 points) Prove that  $\aleph_0 + 3 = \aleph_0$  by constructing a suitable bijection.

**Task 5.** (20 points) Construct a DFA for the language

$$L = L_1 \cup L_2$$

where  $L_1 = (a \cup b)^*ab$  and  $L_2 = (aa \cup ab)^*$ .