Formale Systeme

Example Test 1

Task 1. (25 points) Prove that the following propositional formula is a tautology in at least two different ways (via a truth table, or via a calculation, or via a derivation):

$$\neg(A \wedge B) \wedge B \Rightarrow \neg A$$

Task 2. (15 points) Show that the following formula is not a tautology:

$$(A \Rightarrow C) \Rightarrow (A \Rightarrow B) \wedge (B \Rightarrow C)$$

Task 3. (10 points) Let $\mathbb{P}$ denote the set of all prime (natural) numbers. Write the following statement as a predicate formula:

Every prime number is a sum of two other prime numbers.

Task 4. (15 points) Let X be a set in $\mathcal{P}(U)$ and let C be a family of so-called closed sets, $C \subseteq \mathcal{P}(U)$. The *closure* of the set X, denoted by $\overline{X}$ is the smallest closed set that contains X.

Write a predicate formula that exactly defines $\overline{X}$.
(Note that X is free in this formula.)

Task 5. (20 points) Write an abstract predicate formula corresponding to the following syllogism example, and prove that it is a tautology.

No reptiles have fur.
All snakes are reptiles.

Therefore, no snakes have fur.

Task 6. (20 points) Prove that for arbitrary sets A and B, $(A \cup B) \cap A = A$.