Strengthening
and
weakening

We had

Definition: Two abstract propositions P and Q are equivalent, notation $P \stackrel{val}{=} Q$, iff

(1) Always when P has truth value 1, also Q has truth value 1, and

(2) Always when Q has truth value 1, also P has truth value 1.

if we relax this, we get strengthening

Strengthening

Definition: The abstract proposition P is stronger than Q, notation $P \overset{val}{\models} Q$, iff

~~(1)~~ Always when P has truth value 1, also Q has truth value 1~~, and~~

~~(2) Always when Q has truth value 1, also P has truth value 1.~~

Q is weaker than P

Strengthening

Definition: The abstract proposition P is stronger than Q, notation $P \models_{val} Q$, iff
always when P has truth value 1,
also Q has truth value 1.

always when P is true,
Q is also true

Q is weaker than P

Properties

Lemma E1: $P \overset{val}{=} Q$ iff $P \Leftrightarrow Q$ is a tautology.

Lemma EW1: $P \overset{val}{=} Q$ iff $P \overset{val}{\models} Q$ and $Q \overset{val}{\models} P$.

Lemma W2: $P \overset{val}{\models} P$

Lemma W3: If $P \overset{val}{\models} Q$ and $Q \overset{val}{\models} R$ then $P \overset{val}{\models} R$

Lemma W4: $P \overset{val}{\models} Q$ iff $P \Rightarrow Q$ is a tautology.

Standard Weakenings

and-or-weakening

$$P \wedge Q \overset{val}{\models} P$$

$$P \overset{val}{\models} P \vee Q$$

Extremes

$$\mathrm{F} \overset{val}{\models} P$$

$$P \overset{val}{\models} \mathrm{T}$$

Calculating with weakenings

(the use of standard weakenings)

Substitution

just holds

Simple

$$\frac{\phi \overset{val}{\models} \psi}{\phi\{\xi/P\} \overset{val}{\models} \psi\{\xi/P\}}$$

Sequential

$$\frac{\phi \overset{val}{\models} \psi}{\phi[\xi/P][\eta/Q] \overset{val}{\models} \psi[\xi/P][\eta/Q]}$$

Simultaneous

$$\frac{\phi \overset{val}{\models} \psi}{\phi[\xi/P, \eta/Q] \overset{val}{\models} \psi[\xi/P, \eta/Q]}$$

EVERY occurrence of P is substituted!

The rule of Leibniz

does not hold for weakening!

Leibniz

$$\frac{\phi \overset{val}{\models} \psi}{C[\phi] \overset{val}{\models} C[\psi]}$$

formula that has ϕ as a sub formula

The rule of Leibnitz

does not hold for weakening!

Leibnitz

$$\frac{\phi \overset{val}{\models} \psi}{C[\phi] \overset{val}{\models} C[\psi]}$$

Monotonicity

$$\frac{P \overset{val}{\models} Q}{P \wedge R \overset{val}{\models} Q \wedge R}$$

$$\frac{P \overset{val}{\models} Q}{P \vee R \overset{val}{\models} Q \vee R}$$