



LOGIC and SETS

# Formale Systeme

3VO + 2PS

**Lecturer:** Dr. Ana Sokolova

**Instructions:** Dr. Ana Sokolova + Markus Flatz

[http://cs.uni-salzburg.at/~anas/Ana\\_Sokolova/FormaleSysteme2017/](http://cs.uni-salzburg.at/~anas/Ana_Sokolova/FormaleSysteme2017/)

# The Rules of the Game

- **Lectures** Wednesday 1:15 pm - 2 pm in T01  
Thursday 10:15 am - 12 am in T01

- **Instructions**

Group 1, Thursday 1:15 pm - 3 pm (AS) in T01

Group 2, Thursday 3:15 pm - 5 pm (AS) in T01

Group 3, Thursday 1:15 pm - 3 pm (MF) in T03

- **Tutors** Simon Bauer and Philipp Mayer  
Tuesday 4 pm - 5:30 pm in T??

- **Books**

Logical Reasoning: A First Course by R. Nederpelt and F. Kamaraddine

How to Think Like a Mathematician by K. Houston

Modellierung: Grundlagen und formale Methoden by U. Kastens and H. Kleine Büning

Introduction to Automata Theory, Languages, and Computation by J. E. Hopcroft, R. Motwani and J.D. Ullman



starting  
in two weeks

# The Rules... Instructions

## (PS)

- **Instruction exercises** on the web  
[http://cs.uni-salzburg.at/~anas/Ana\\_Sokolova/FormaleSystemeProseminar2017/](http://cs.uni-salzburg.at/~anas/Ana_Sokolova/FormaleSystemeProseminar2017/)  
on Thursday afternoons
- To be solved by the students (ideally alone)
- In class we will have a small **test** every week except the first week (**1 simple exercise**)  
and then present solutions/discuss the exercises (**sometimes students will be asked to present**)

# The Rules... Instructions (PS)

- The test exercise will be graded each week
- The graded exercise will be returned to you in class (with feedback) one week later
- **Grade** based on
  - (1) the grades of the test exercises and
  - (2) activity in class (ability to present solutions)
- All information about the course / rules / exams / grading is / will be on the course webpage

# The Rules... Exam (VO)

- Written exams
- **Written exam** in February, April, and July or **two partial tests during the semester**
- Grade based on the # of points on the written exam (or sum of the points on the partial tests)
- For better grade **oral exam** after the written one **upon appointment**
- You can pass the course if you have 55% of the maximal points on the exam.

# The Rules... Tests (VO)

- One test end of November, one beginning of February
- The tests are **partial** (half material)
- You can pass via tests if the sum of your points on both tests is at least 55% of the sum of maximal points on the tests **and** if on each test you have at least 20% of the maximal points
- The tests and the exams consist of exercises / questions related to the material taught in class

# Some advice

- It starts easy, but soon it gets more difficult
- There accumulates lots of material for the exam
- **Best is** to **regularly** study, practice, solve the exercises **yourself!**

# ECTS points

1 ECTS = 25 hours

- 3 ECTS for the lectures (VO)
- 4 ECTS for the instructions (PS)

$7 \times 25 = 175$  hours

in class app. 52 hours

123 hours remain for  
your studying !

better: 1 week before exam (40  
hours) + 6.3 hours a week

app, 8.8 hours a  
week :-)



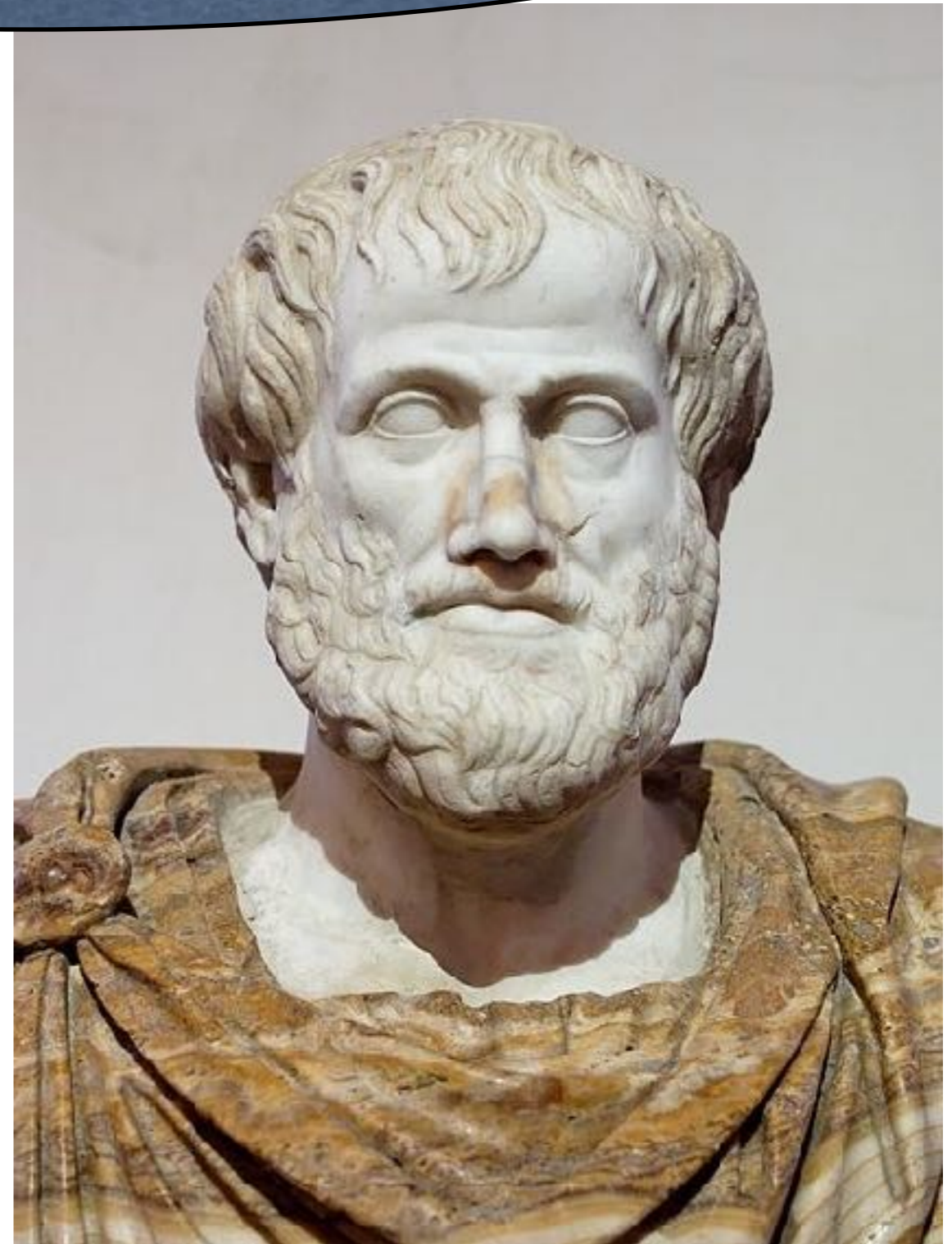
Logic = study of correct reasoning

In the beginning

Aristotle +/- 350 B.C.

Organon

19 syllogisms



# Formal Logic

Gottfried Wilhelm Leibnitz  
(1646 - 1716)

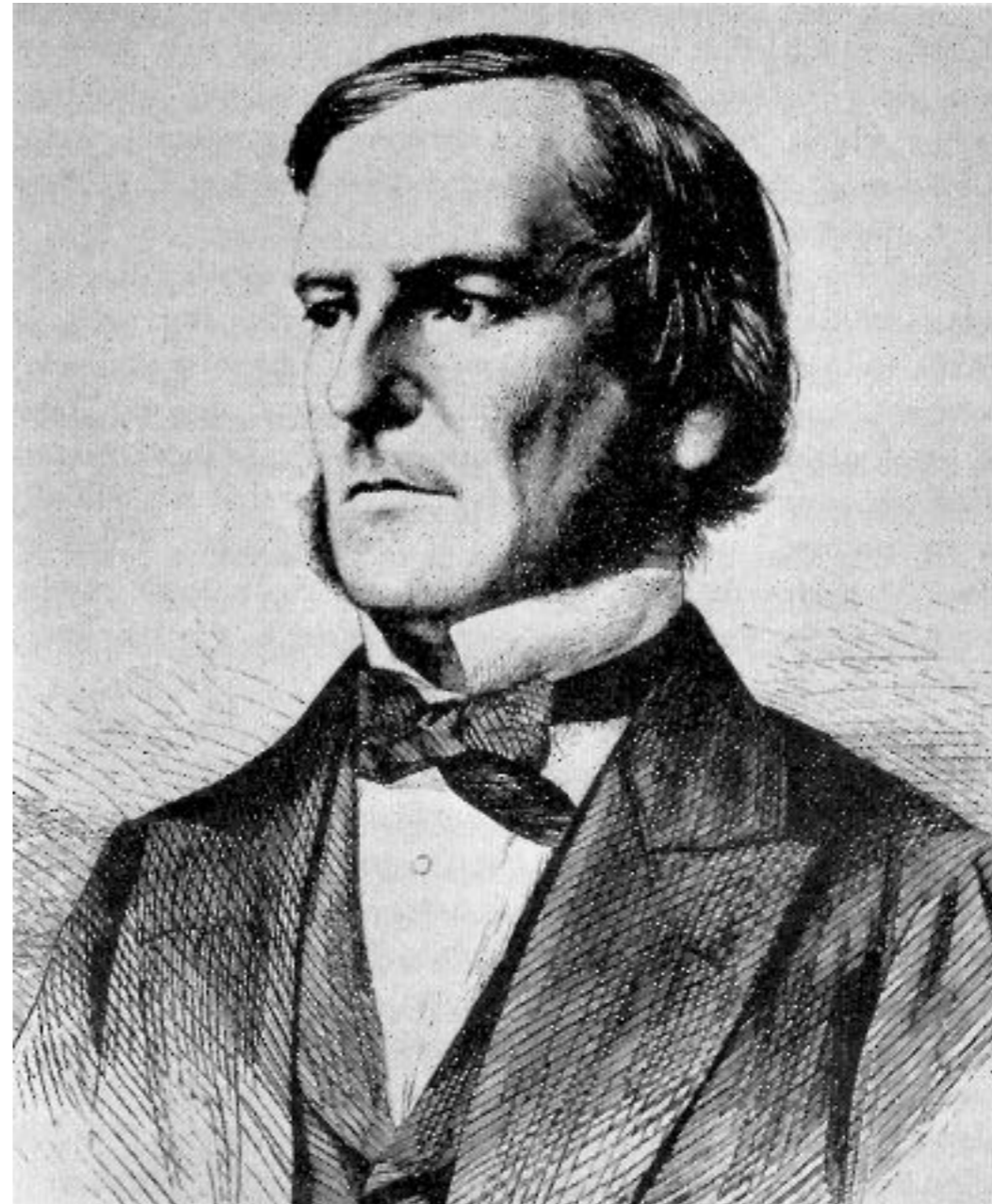
Beginnings of *symbolic logic*



# Boolean Logic

George Boole  
(1815 - 1864)

Boolean logic



# We will learn

Starting this week

- **Naive Set Theory** - sets, relations, mappings, numbers and structures, ordered sets
- **Logical Calculations** - propositional logic, predicate logic
- **Logical Derivations** - reasoning
- **Basics of formal models** - finite automata, transition systems, graphs, grammars...

# Why formal models/ methods?

- For better understanding of a complex system, problem, task,... models, **abstractions** are needed
- For rigorous precise **reasoning** about a complex system, problem, task

# The river-crossing puzzle

- A man stands with a wolf, a goat, and a cabbage at the left bank of a river, that he wants to cross.
- The man has a boat that is large enough to carry him and another object to the other side.
- If the man leaves the wolf and the goat, or the goat and the cabbage on one side without supervision, one of them will get eaten :-)
- Is it possible to cross the river so that neither the goat nor the cabbage is eaten?

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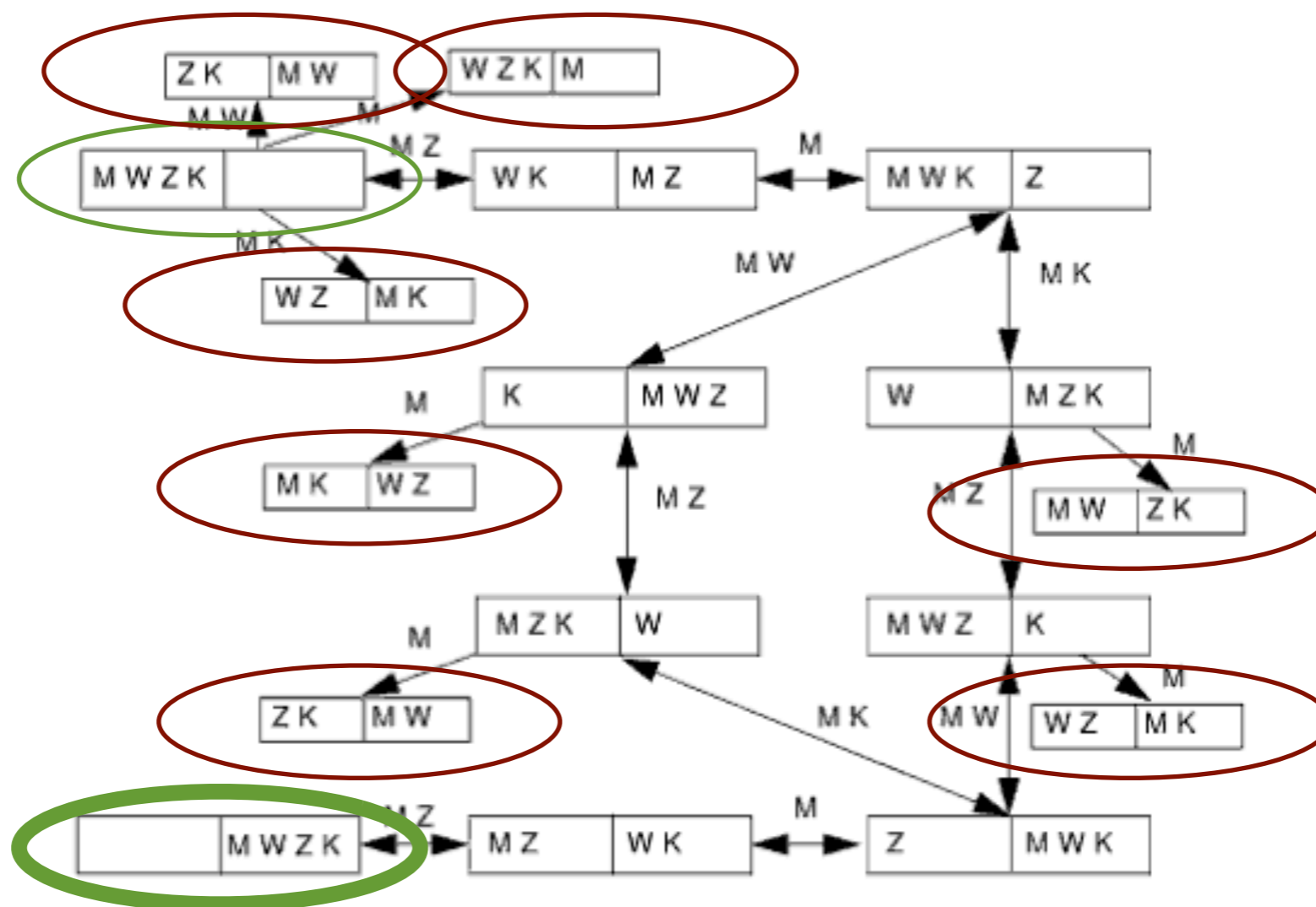


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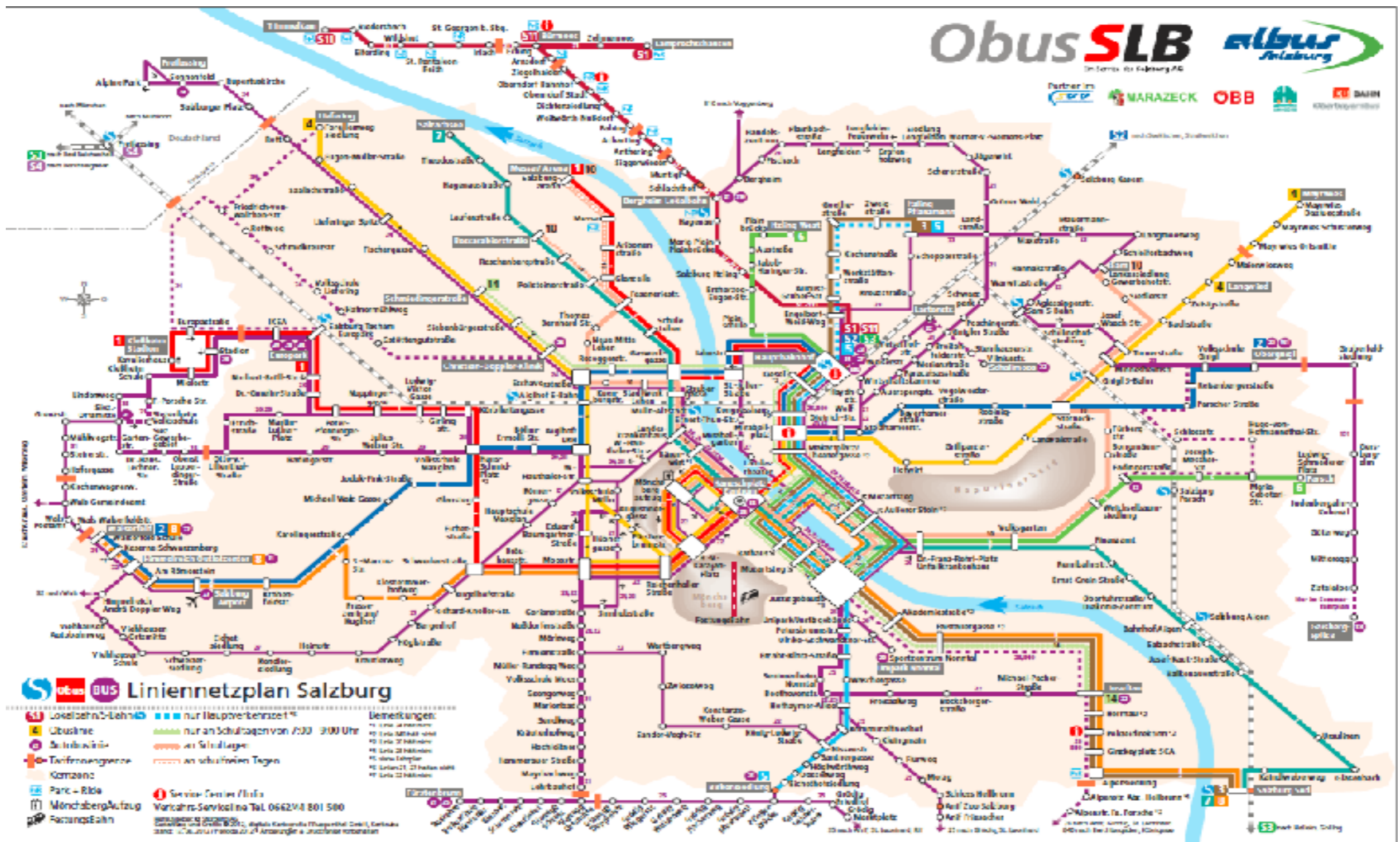
# The river-crossing puzzle

Formalization with a finite automaton [Kastens et al.] :



states and transitions

# Another model example



# Sets

- A **set**  $S$  is a collection of different objects, the elements of  $S$
- We write  $x \in S$  for 'x is an element of S'
- A set 'can' be specified by
  - (1) listing its elements, e.g.  $S = \{1, 3, 7, 18\}$
  - (2) **specifying a property**, e.g.  $S = \{x \mid P(x)\}$
- Sets can be **finite** e.g.  $\{\clubsuit, \heartsuit\}$  or **infinite** e.g.  $\mathbb{N}$
- The set with no elements is the **empty set**, notation  $\emptyset$
- The 'number' of elements in a set  $S$  is the **cardinality** of  $S$ , notation  $|S|$

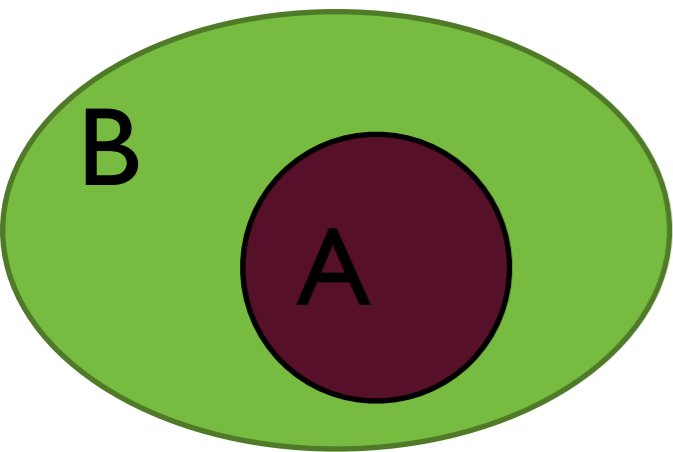
$P$  is a proposition over  $x$ , which is true or false

# Sets - properties

- All elements of a set are different
- The elements of a set are not ordered
- The same set can be specified in different ways, e.g.  
 $\{1,2,3,4\}$ ,  $\{2,3,1,4\}$ ,  $\{i \mid i \in \mathbb{N} \text{ and } 0 < i < 5\}$

# Subsets, equality

**Def.**  $A \subseteq B$  iff all elements of  $A$  are elements of  $B$   
[iff for all  $a$ , if  $a \in A$ , then  $a \in B$ ]



[iff  $\forall a (a \in A \Rightarrow a \in B)$ ]

logical formula

quantifier

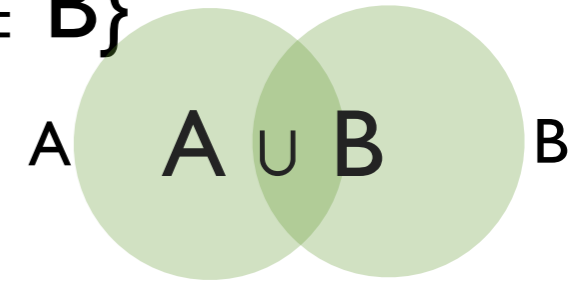
logical  
connective

**Def.**  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$

**Def.**  $A \subset B$  iff  $A \subseteq B$  and  $A \neq B$

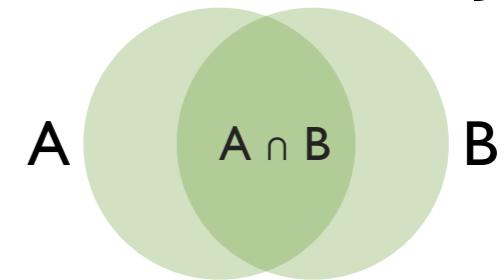
# Operations on sets

Def. Union (Vereinigung)  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

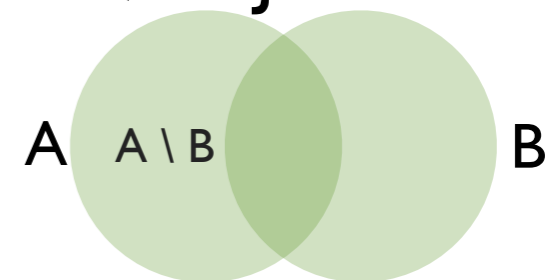


Def. Intersection (Durchschnitt)  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

A and B are **disjoint** if  $A \cap B = \emptyset$



Def. Difference (Differenz)  $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



Def. Direct product (Kartesisches Produkt)

$(A \times B) \times C \neq A \times (B \times C)$

$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

ordered pairs

Def. Powerset (Potenzmenge)  $\mathcal{P}(A) = \{X \mid X \subseteq A\}$

# Russell's paradox

- Let  $P$  be the set of all sets that are not an element of itself
- Hence,  $P = \{x \mid x \notin x\}$
- Is  $P \in P$  ?
- **Contradiction!**

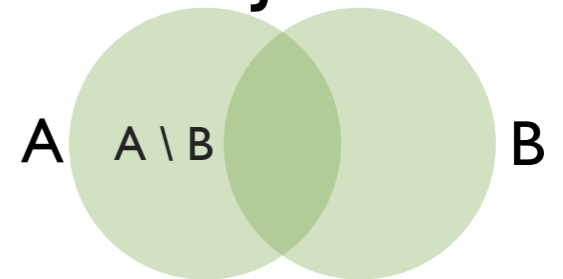
The need for a universal set  $U$

$$S = \{x \mid x \in U \text{ and } P(x)\}$$



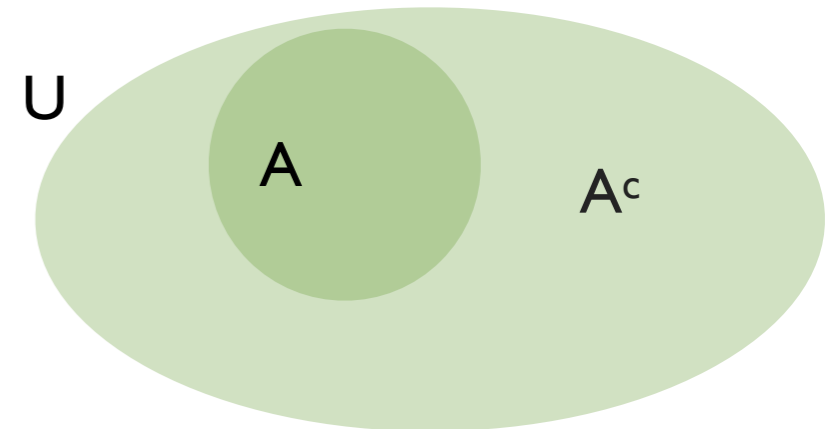
# Operations on sets

Def. Difference (Differenz)  $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



Given a universal set U

Def. Complement (Komplement)  $A^c = \{x \mid x \in U \text{ and } x \notin A\}$



Hence  $A^c = U \setminus A$

# Properties of sets

1.  $\emptyset \subseteq X$

2. If  $A \subseteq B$  and  $B \subseteq C$ , then  $A \subseteq C$

3.  $X \cap Y \subseteq X$ ,  $X \cap Y \subseteq Y$

4.  $X \subseteq X \cup Y$ ,  $Y \subseteq X \cup Y$

5. If  $X' \subseteq Y'$  and  $X'' \subseteq Y''$ , then  $X' \cap X'' \subseteq Y' \cap Y''$

6. If  $X' \subseteq Y'$  and  $X'' \subseteq Y''$ , then  $X' \cup X'' \subseteq Y' \cup Y''$

7.  $X \cap Y = X$  iff  $X \subseteq Y$

8.  $X \cap X = X$  (idempotence)

9.  $X \cup X = X$  (idempotence)

10.  $X \cap \emptyset = \emptyset$

# Properties of sets

11.  $X \cup \emptyset = X$

12.  $X \cap Y = Y \cap X$  (commutativity)

13.  $X \cup Y = Y \cup X$  (commutativity)

14.  $X \cap (Y \cap Z) = (X \cap Y) \cap Z$  (associativity)

15.  $X \cup (Y \cup Z) = (X \cup Y) \cup Z$  (associativity)

16.  $X \cap (X \cup Y) = X$  (absorption)

17.  $X \cup (X \cap Y) = X$  (absorption)

18.  $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$  (distributivity)

19.  $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$  (distributivity)

20.  $X \setminus Y \subseteq X$

# Properties of sets

$$21. (X \setminus Y) \cap Y = \emptyset$$

$$22. X \cup Y = X \cup (Y \setminus X)$$

$$23. X \setminus X = \emptyset$$

$$24. X \setminus \emptyset = X$$

$$25. \emptyset \setminus X = \emptyset$$

$$26. \text{If } X \subseteq Y, \text{ then } X \setminus Y = \emptyset$$

$$27. (X^c)^c = X$$

$$28. (X \cap Y)^c = X^c \cup Y^c \quad (\text{De Morgan})$$

$$29. (X \cup Y)^c = X^c \cap Y^c \quad (\text{De Morgan})$$

$$30. X \times \emptyset = \emptyset \quad \emptyset \times X = \emptyset$$

$$31. \emptyset \times X = \emptyset$$

$$32. \text{If } X \subseteq Y, \text{ then } \mathcal{P}(X) \subseteq \mathcal{P}(Y)$$