LOGIC and SETS

Formale Systeme

3VO + 2PS

Lecturer: Dr. Ana Sokolova

Instructions: Dr. Ana Sokolova + Sebastian Arming + Markus Flatz

http://cs.uni-salzburg.at/~anas/Ana_Sokolova/FormaleSysteme2016/

The Rules of the Game

- **Lectures** Wednesday 1:15 pm - 2 pm in T01
  Thursday 10:15 am - 12 am in T01

- **Instructions**
  Group 1, Thursday 1:15 pm - 3 pm (AS) in T01
  Group 2, Thursday 1:15 pm - 3 pm (SA) in T02
  Group 3, Thursday 1:15 pm - 3 pm (MF) in T03

  starting next week

- **Tutors** Michael Moser and Sarah Sophie Sallinger
  Tuesday 5 pm - 6:30 pm in T??

- **Books**
  Logical Reasoning: A First Course by R. Nederpelt and F. Kamaraddine

  Modellierung: Grundlagen und formale Methoden by U. Kastens and H. Kleine Büning

  Introduction to Automata Theory, Languages, and Computation by J. E. Hopcroft, R. Motwani and J.D. Ullman

The Rules... Instructions (PS)

- Instruction exercises on the web
  http://cs.uni-salzburg.at/~anas/Ana_Sokolova/FormaleSystemeProseminar2016/
  on Thursday afternoons
- To be solved by the students (ideally alone)
- In class we will have a small test every week except the first week (1 simple exercise)
  and then present solutions/discuss the exercises
  (sometimes students will be asked to present)

The Rules... Instructions (PS)

- The test exercise will be graded each week
- The graded exercise will be returned to you in class (with feedback) one week later
- Grade based on
  (1) the grades of the test exercises and
  (2) activity in class (ability to present solutions)
- All information about the course / rules / exams / grading is / will be on the course webpage

The Rules... Exam (VO)

- Written exams
- Written exam in February, April, and July
  or two partial tests during the semester
- Grade based on the # of points on the written exam (or sum of the points on the partial tests)
- For better grade oral exam after the written one upon appointment
- You can pass the course if you have 55% of the maximal points on the exam.

The Rules... Tests (VO)

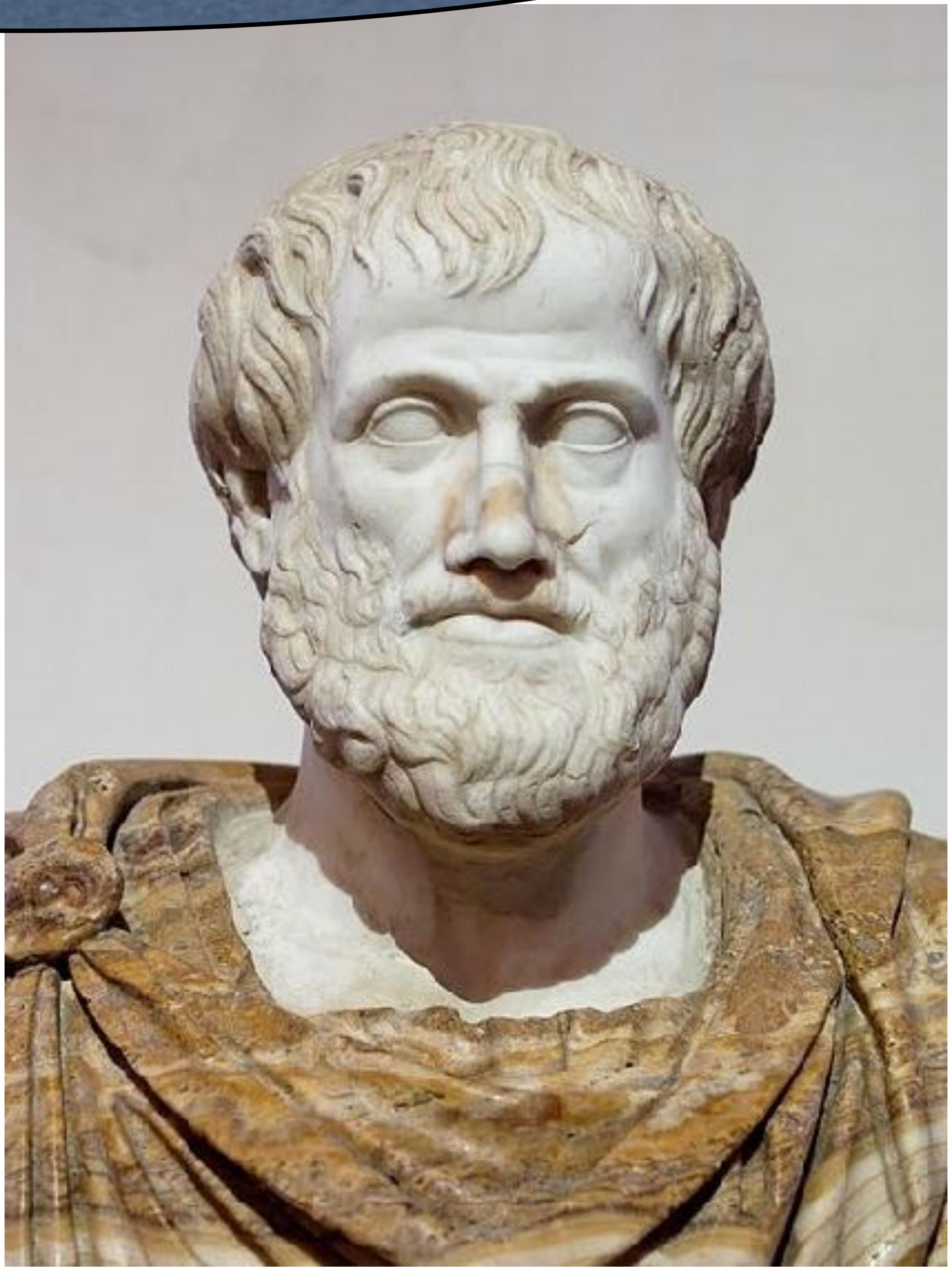
- One test end of November, one beginning of February
- The tests are partial (half material)
- You can pass via tests if the sum of your points on both tests is at least 55% of the sum of maximal points on the tests and if on each test you have at least 20% of the maximal points
- The tests and the exams consist of exercises / questions related to the material taught in class

Some advice

- It starts easy, but soon it gets more difficult
- There accumulates lots of material for the exam
- Best is to regularly study, practice, solve the exercises yourself!

Logic = study of correct reasoning

In the beginning



Aristotle +/- 350 B.C.

Organon

19 syllogisms

Formal Logic

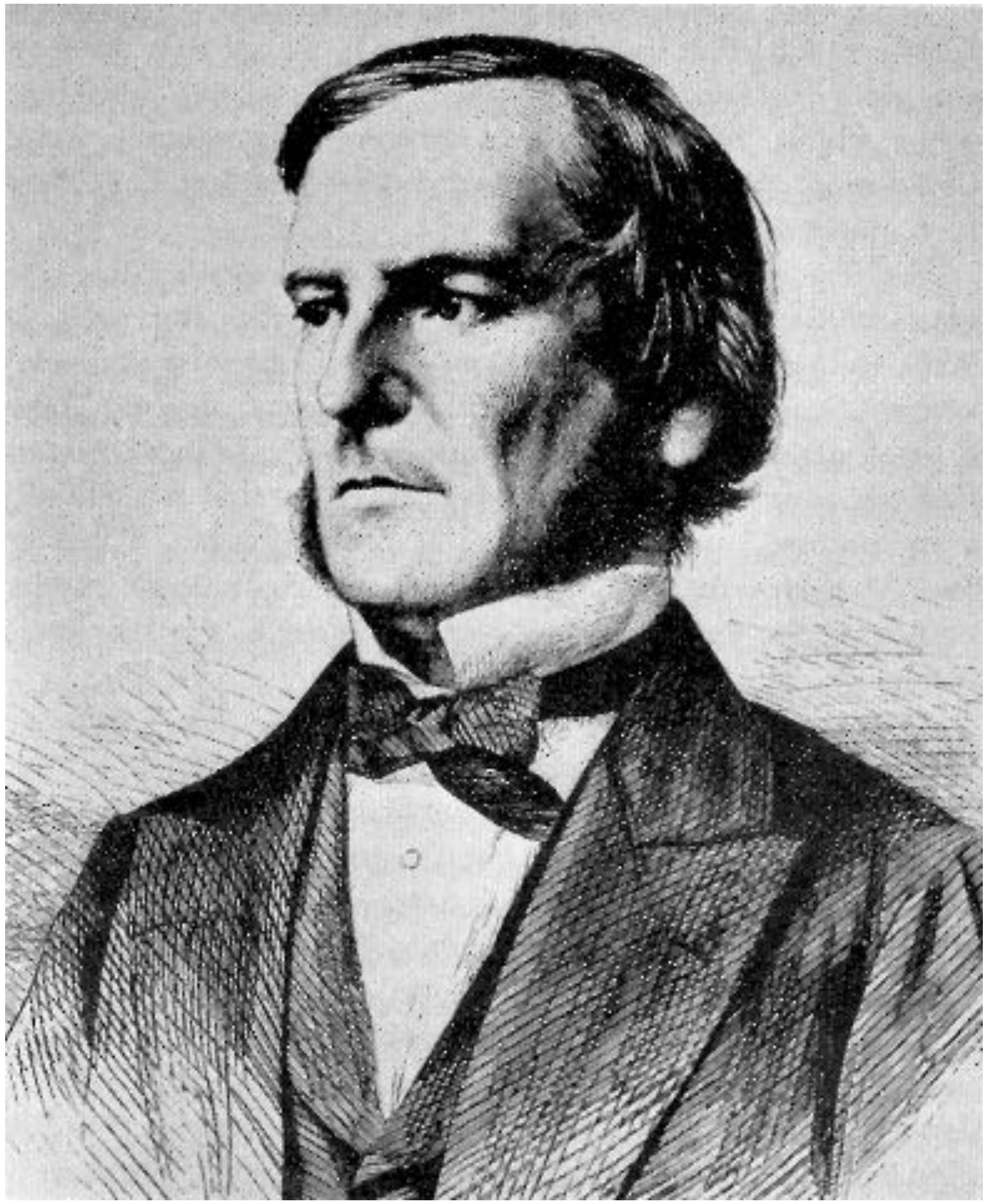
Gottfried Wilhelm Leibnitz
(1646 - 1716)

Beginnings of symbolic logic

Boolean Logic

George Boole
(1815 - 1864)

Boolean logic



We will learn

Starting this week

- Naive Set Theory - sets, relations, mappings, numbers and structures, ordered sets
- Logical Calculations - propositional logic, predicate logic
- Logical Derivations - reasoning
- Basics of formal models - finite automata, transition systems, graphs, grammars...

Why formal models/ methods?

- For better understanding of a complex system, problem, task,... models, abstractions are needed
- For rigorous precise reasoning about a complex system, problem, task

The river-crossing puzzle

- A man stands with a wolf, a goat, and a cabbage at the left bank of a river, that he wants to cross.
- The man has a boat that is large enough to carry him and another object to the other side.
- If the man leaves the wolf and the goat, or the goat and the cabbage on one side without supervision, one of them will get eaten :-(
- Is it possible to cross the river so that neither the goat nor the cabbage is eaten?

[Hopcroft et al, Kastens et al]

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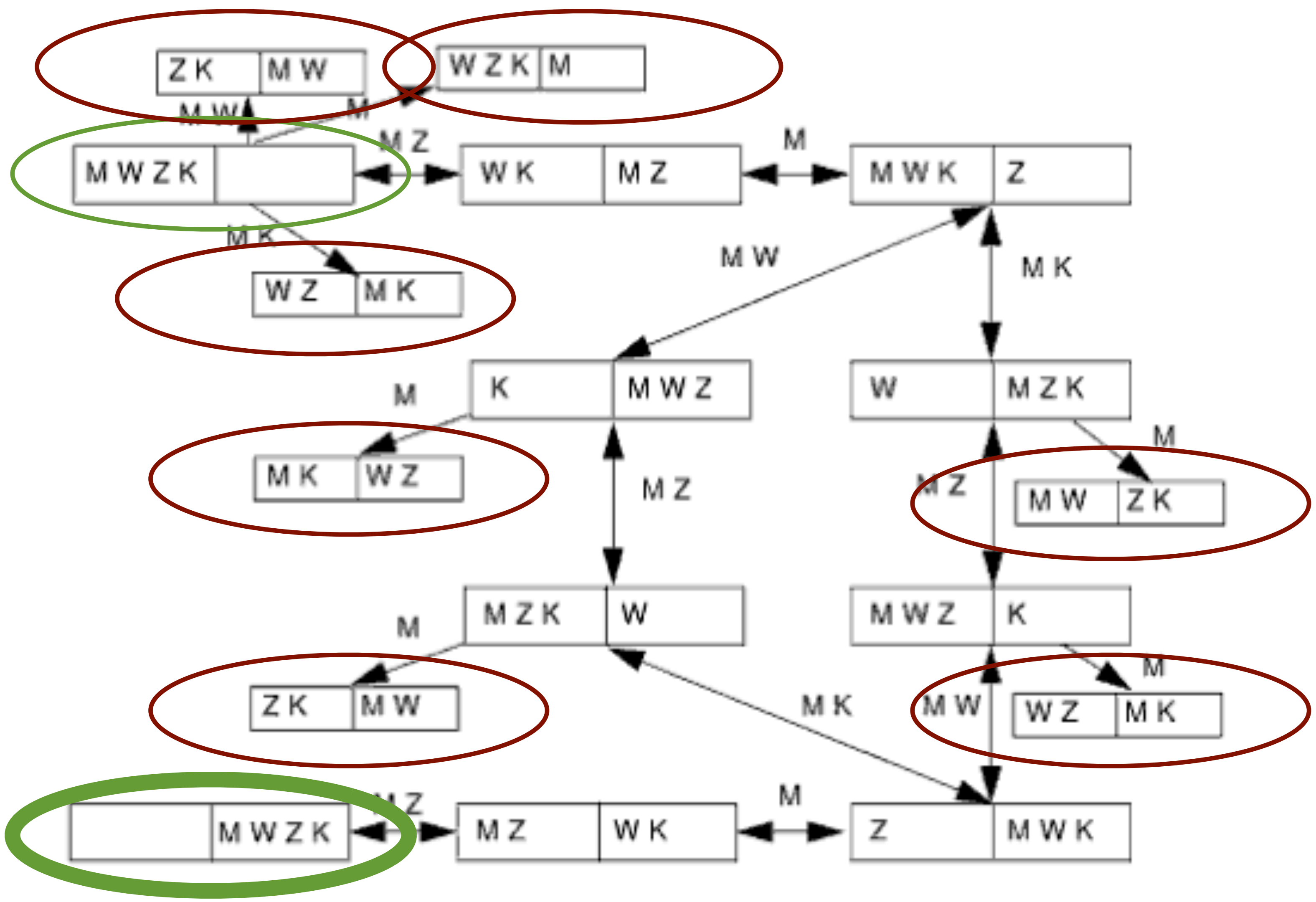
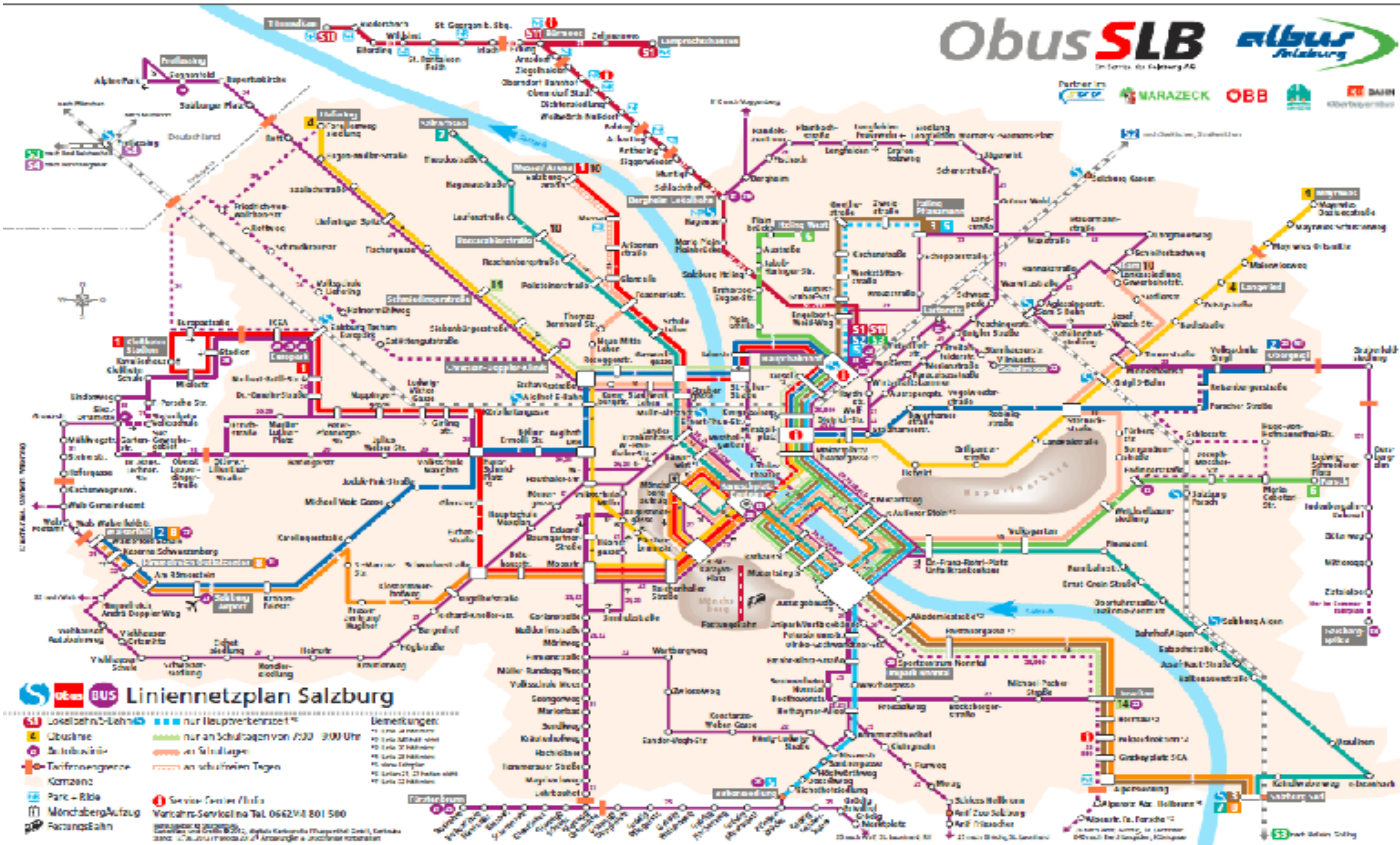
The river-crossing puzzle

Formalization with a finite automaton [Kastens et al.] :



states and transitions

Another model example



Sets

- A set S is a collection of different objects, the elements of S

- We write $x \in S$ for `x is an element of S'

- A set `can' be specified by
  (1) listing its elements, e.g. S = {1,3,7,18}
  (2) specifying a property, e.g. S = {x | P(x)}

  P is a proposition over x, which is true or false

- Sets can be finite e.g. {♣, ♥} or infinite e.g. $\mathbb{N}$

- The set with no elements is the empty set, notation $\varnothing$

- The `number' of elements in a set S is the cardinality of S, notation |S|

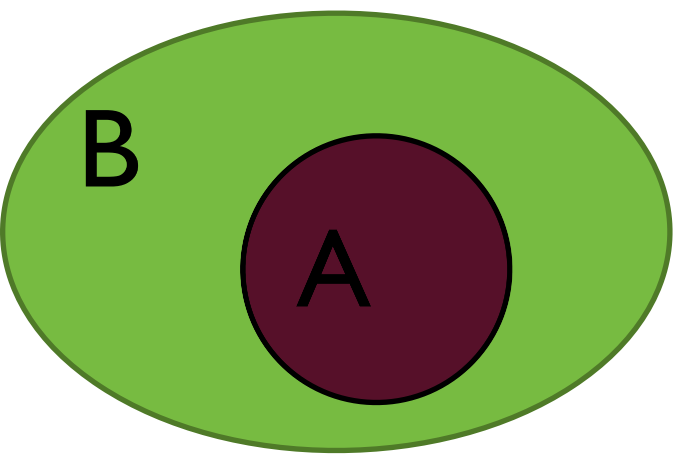
Sets - properties

- All elements of a set are different
- The elements of a set are not ordered
- The same set can be specified in different ways, e.g. $\{1,2,3,4\}$, $\{2,3,1,4\}$, $\{i \mid i \in \mathbb{N} \text{ and } 0 < i < 5\}$

Subsets, equality

Def. $A \subseteq B$ iff all elements of A are elements of B

[iff for all a, if $a \in A$, then $a \in B$]



[iff $\forall a\ (a \in A \Rightarrow a \in B)$]

logical formula

quantifier

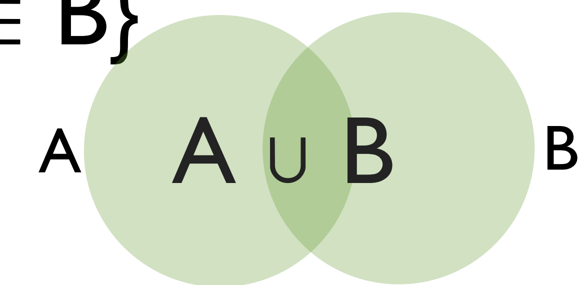
logical connective

Def. $A = B$ iff $A \subseteq B$ and $B \subseteq A$

Def. $A \subset B$ iff $A \subseteq B$ and $A \neq B$

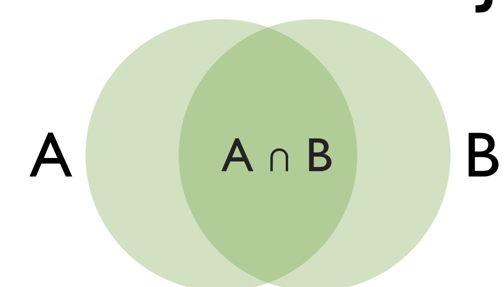
Operations on sets

Def. Union (Vereinigung) $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

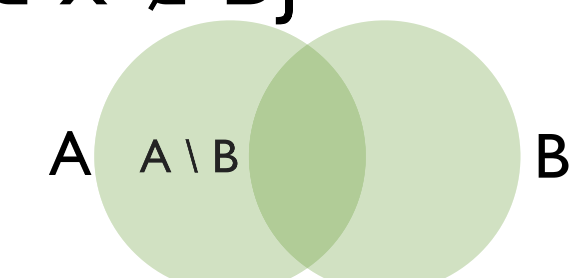


Def. Intersection (Durchschnitt) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

A and B are **disjoint** if $A \cap B = \varnothing$



Def. Difference (Differenz) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



Def. Direct product (Kartesisches Produkt)

$(A \times B) \times C \neq A \times (B \times C)$

$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

ordered pairs

Def. Powerset (Potenzmenge) $\mathcal{P}(A) = \{X \mid X \subseteq A\}$

Russell's paradox

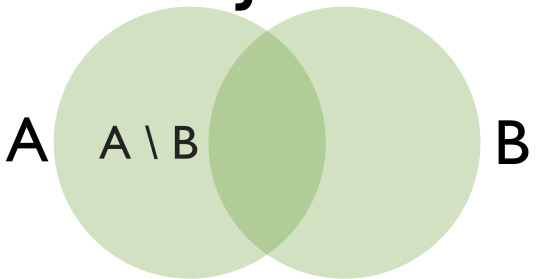
- Let P be the set of all sets that are not an element of itself
- Hence, $P = \{ x \mid x \notin x \}$
- Is $P \in P$?
- Contradiction!

The need for a universal set U

$S = \{x \mid x \in U \text{ and } P(x)\}$

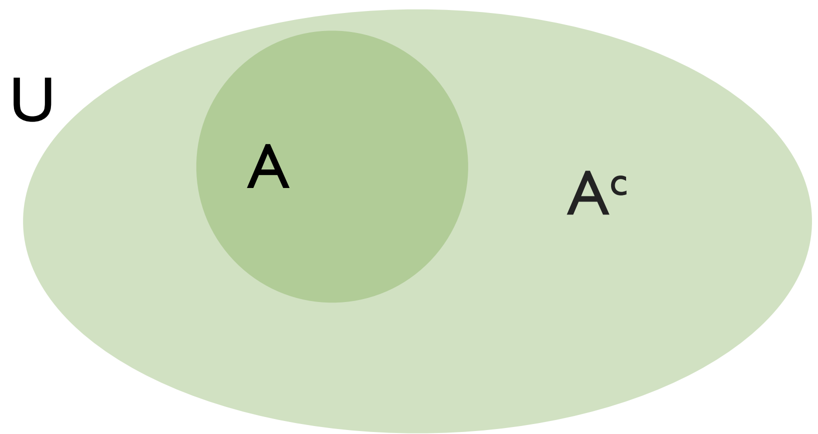
Operations on sets

Def. Difference (Differenz) $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$



Given a universal set U

Def. Complement (Komplement) $A^c = \{x \mid x \in U \text{ and } x \notin A\}$



Hence $A^c = U \setminus A$

Properties of sets

1. $\emptyset \subseteq X$
2. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
3. $X \cap Y \subseteq X$, $X \cap Y \subseteq Y$
4. $X \subseteq X \cup Y$, $Y \subseteq X \cup Y$
5. If $X' \subseteq Y'$ and $X'' \subseteq Y''$, then $X' \cap X'' \subseteq Y' \cap Y''$
6. If $X' \subseteq Y'$ and $X'' \subseteq Y''$, then $X' \cup X'' \subseteq Y' \cup Y''$
7. $X \cap Y = X$ iff $X \subseteq Y$
8. $X \cap X = X$ (idempotence)
9. $X \cup X = X$ (idempotence)
10. $X \cap \emptyset = \emptyset$

Properties of sets

11.	$X \cup \varnothing = X$
12.	$X \cap Y = Y \cap X$ (commutativity)
13.	$X \cup Y = Y \cup X$ (commutativity)
14.	$X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (associativity)
15.	$X \cup (Y \cup Z) = (X \cup Y) \cup Z$ (associativity)
16.	$X \cap (X \cup Y) = X$ (absorption)
17.	$X \cup (X \cap Y) = X$ (absorption)
18.	$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ (distributivity)
19.	$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ (distributivity)
20.	$X \setminus Y \subseteq X$

Properties of sets

21.	$(X \setminus Y) \cap Y = \varnothing$
22.	$X \cup Y = X \cup (Y \setminus X)$
23.	$X \setminus X = \varnothing$
24.	$X \setminus \varnothing = X$
25.	$\varnothing \setminus X = \varnothing$
26.	If $X \subseteq Y$, then $X \setminus Y = \varnothing$
27.	$(X^c)^c = X$
28.	$(X \cap Y)^c = X^c \cup Y^c$ (De Morgan)
29.	$(X \cup Y)^c = X^c \cap Y^c$ (De Morgan)
30.	$X \times \varnothing = \varnothing$ $\varnothing \times X = \varnothing$
31.	$\varnothing \times X = \varnothing$
32.	If $X \subseteq Y$, then $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$