

Properties of sets

1. $\emptyset \subseteq X$

2. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

3. $X \cap Y \subseteq X$, $X \cap Y \subseteq Y$

4. $X \subseteq X \cup Y$, $Y \subseteq X \cup Y$

5. If X

6. If X

7. $X \cap Y = X$ iff $X \subseteq Y$

8. $X \cap X = X$ (idempotence)

9. $X \cup X = X$ (idempotence)

10. $X \cap \emptyset = \emptyset$

Properties of sets

11. $X \cup \emptyset = X$

12. $X \cap Y = Y \cap X$ (commutativity)

13. $X \cup Y = Y \cup X$ (commutativity)

14. $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (associativity)

15. $X \cup (Y \cup Z) = (X \cup Y) \cup Z$ (associativity)

16. $X \cap (X \cup Y) = X$ (absorption)

17. $X \cup (X \cap Y) = X$ (absorption)

18. $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ (distributivity)

19. $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ (distributivity)

20. $X \setminus Y \subseteq X$

Properties of sets

$$21. (X \setminus Y) \cap Y = \emptyset$$

$$22. X \cup Y = X \cup (Y \setminus X)$$

$$23. X \setminus X = \emptyset$$

$$24. X \setminus \emptyset = X$$

$$25. \emptyset \setminus X = \emptyset$$

$$26. \text{If } X \subseteq Y, \text{ then } X \setminus Y = \emptyset$$

$$27. (X$$

$$28. (X$$

$$29. (X$$

$$30. X \times \emptyset = \emptyset$$

$$31. \emptyset \times X = \emptyset$$

$$32. \text{If } X \subseteq Y, \text{ then } \mathcal{P}(X) \subseteq \mathcal{P}(Y)$$

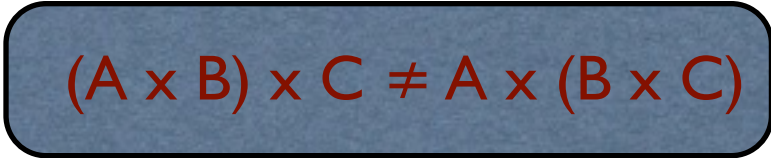
Product of multiple sets

Direct product (Kartesisches Produkt)

$$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$$



ordered pairs


$$(A \times B) \times C \neq A \times (B \times C)$$

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In general, for sets A_1, A_2, \dots, A_n with $n \geq 1$,

$$A_1 \times A_2 \times \dots \times A_n = \prod_{1 \leq i \leq n} A_i = \{(x_1, x_2, \dots, x_n) \mid x_i \in A_i \text{ for } 1 \leq i \leq n\}$$

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if $A_i = A$ for all i ,
then the product is
denoted A^n

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Finite sequences, words

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Let A be a set, an alphabet

$$A^0 = \{\varepsilon\}$$

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Relations

Def. If A and B are sets, then any subset $R \subseteq A \times B$ is a (binary) relation between A and B

Def. R is a relation on A if $R \subseteq A \times A$

some relations are special

Relations

Def. If A and B are sets, then any subset $R \subseteq A \times B$ is a (binary) relation between A and B

similarly, unary relation (subset), n-ary relation...

Def. R is a relation on A if $R \subseteq A \times A$

some relations are special

Special relations

A relation $R \subseteq A \times A$ is:

reflexive	iff	for all $a \in A$, $(a,a) \in R$
symmetric	iff	for all $a,b \in A$, if $(a,b) \in R$, then $(b,a) \in R$
transitive	iff	for all $a,b,c \in A$, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$
irreflexive	iff	for all $a \in A$, $(a,a) \notin R$
antisymmetric	iff	for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$
asymmetric	iff	for all $a,b \in A$, if $(a,b) \in R$, then $(b,a) \notin R$
total	iff	for all $a,b \in A$, $(a,b) \in R$ or $(b,a) \in R$

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(infix) notation aRb for $(a,b) \in R$