

Formale Systeme

Example tasks for Test 2, 2015

Task 1. (15) Write down the following statement as a predicate formula:

There exists a 3-element subset of natural numbers that has a 2-element subset whose sum of elements is not divisible by 2.

Is this statement true? (No detailed proof is required, just some intuitive explanation.)

Task 2. (15 + 5) Prove that the following formula is a tautology:

$$(\forall x[D(x) : P(x)] \wedge \neg\exists y[D(y) : P(y)]) \Rightarrow \neg\exists z[D(z) : T].$$

Using that, show that the following statement is true: If all sheep are white and there is no white sheep, then there is no sheep.

Task 3. (10 + 10)

(a) Let X be a set and let $1 = \{*\}$. Show that $X \sim \{f \mid f: 1 \rightarrow X\}$.

(b) Prove that $\aleph_0 + \aleph_0 = \aleph_0$.

Task 4. (20) Let n be any natural number that is larger than or equal to 1. Prove (by induction) that then $3^n > 2^n$.

[Recall the inductive definition of k^n for natural numbers k, n : $k^0 = 1$; $k^{n+1} = k^n \cdot k$.]

Task 5. (10 + 10) Construct an NFA for the language described by the regular expression:

$$(0 \cup 1)^*(11 \cup 101).$$

Determinize your automaton then to obtain a DFA for the given language.

Task 6. (15) Let A be a set. Show that the set P_A of permutations on A , that is

$$P_A = \{f \mid f : A \rightarrow A \text{ is a bijection}\}$$

forms a group with the operation of function composition. How many elements does P_A have if A has n elements?