Formale Systeme

Test 1, Group 2, 7.2.2014

Task 1. (15 + 5) Write down the following statement as a predicate formula:

There exists a 3-element subset of natural numbers that has a 2-element subset whose sum of elements is not divisible by 2.

Is this statement true? (No detailed proof is required, just some intuitive explanation.)

Task 2. (15 + 5) Prove that the following formula is a tautology:

$$(\forall_x[D(x) : P(x)] \wedge \neg\exists_y[D(y) : P(y)]) \Rightarrow \neg\exists_z[D(z) : T].$$

Using that, show that the following statement is true: If all sheep are white and there is no white sheep, then there is no sheep.

Task 3. (10 + 10)

(a) Let X be a set and let $1 = \{*\}$. Show that $X \sim \{f \mid f\colon 1 \to X\}$.

(b) Prove that $\aleph_0 + \aleph_0 = \aleph_0$.

Task 4. (20) Let n be any natural number that is larger than or equal to 1. Prove (by induction) that then $3^n > 2^n$.

[Recall the inductive definition of k^n for natural numbers k, n: $k^0 = 1; k^{n+1} = k^n \cdot k$.]

Task 5. (10 + 10) Construct a finite automaton for the language:

(a) $L = \{a^i b^i c^i \mid i \in \mathbb{N} \wedge 0 \leq i \leq 3\}$.

(b) Let n be a fixed natural number. Give a general construction of a finite automaton for the language $\hat{L} = \{a^i b^i c^i \mid i \in \mathbb{N} \wedge 0 \leq i \leq n\}$.