

Formale Systeme

Test 1, Group 2, 6.12.2013

Task 1. (4*5 points) Write down the definitions of the following notions:

- (a) The sets A and B are disjoint.
- (b) A relation R is symmetric.
- (c) A relation R is a strict order.
- (d) A function $f: A \rightarrow B$ is surjective.

Task 2. (10 points) Prove that the proposition

$$(A \cap B)^c = A^c \cup B^c$$

holds for all sets A and B . (Hint: The easiest way is with a calculation.)

Task 3. (10 points) Show that the following abstract proposition is a not a contradiction

$$(a \Leftrightarrow b) \Rightarrow (\neg a \wedge b) \vee d \vee T$$

Task 4. (20 points) Let U be a set, and consider the relation α defined on $\mathcal{P}(U)$ by

$$(A, B) \in \alpha \text{ if and only if } A \cap B = A.$$

- (a) Check whether α is an equivalence relation.
- (b) Check whether α is a partial order.

Task 5. (20 points) Let $f: A \rightarrow B$. Prove that f is surjective if and only if for any two functions $g_1, g_2: B \rightarrow C$ it holds that $g_1 \circ f = g_2 \circ f \Rightarrow g_1 = g_2$.

Task 6. (20 points) Prove with a calculation that the following formula is a tautology

$$(P \Leftrightarrow Q) \Leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$$