Formale Systeme

Example Test 1

Task 1. (4*5 points) Write down the definitions of the following notions:

(a) A set A is a subset of a set B.

(b) A relation R is transitive.

(c) A relation R is a partial order.

(d) A function $f\colon A \to B$ is surjective.

Task 2. (20 points) Check whether the proposition

$$A \cap B \subseteq C \Rightarrow A \cup B \subseteq C$$

holds for all sets A, B, and C. If so, then give a proof; if not, then give a counter example.

Task 3. (20 points) Let $f\colon A \to B$. We define the *kernel* of the map f as

$$\ker f = \{(a_1, a_2) \mid f(a_1) = f(a_2)\}.$$

- Show that $\ker f$ is an equivalence relation on A.
- Show that f is injective if and only if $\ker f = \Delta_A$ where as usual $\Delta_A = \{(a, a) \mid a \in A\}$ is the diagonal relation on A.

Task 4. (20 points) Show that the following abstract proposition is a contingency (i.e., neither a tautology nor a contradiction)

$$((a \Leftrightarrow b) \Rightarrow (\neg a \vee c)) \vee d \vee (e \wedge T)$$

Advice: Do not make a full truth table.

Task 5. (20 points) Prove with a calculation that the following two formulas are comparable (i.e., one is stronger than the other or vice-versa)

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \quad \text{and} \quad (\neg P \Rightarrow Q) \Rightarrow R$$