Formale Systeme PS

Exercises, Week 8

Example Test exercises

Task 1. Show that the following abstract proposition is a contingency (i.e., neither a tautology nor a contradiction)

$$((a \Leftrightarrow b) \Rightarrow (\neg a \vee c)) \vee d \vee (e \wedge T)$$

Advice: Do not make a full truth table.

Task 2. Prove with a calculation that the following two formulas are comparable (i.e., one is stronger than the other or vice-versa)

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \quad \text{and} \quad (\neg P \Rightarrow Q) \Rightarrow R$$

Task 3. Show with a counter example that:

$$\exists_k[P \vee Q \,:\, R] \overset{val}{\neq} \neg\forall_k[Q \,:\, \neg R]$$

Hint: Simplify first (one or both sides).

Task 4. Write the following sentence (in quotes below) as a formula with connectives and quantifiers. You may use that $\mathbb{P}$ denotes the set of all prime numbers.

"Every even natural number greater than 4 is the sum of two prime numbers."

Task 5. Check whether the proposition

$$A \cap B \subseteq C \Rightarrow A \cup B \subseteq C$$

holds for all sets A, B, and C. If so, then give a proof; if not, then give a counter example.

Regular exercises from Chapter 10

Task 6. Write the following as a set of the form $\{... \mid ...\}$ and also with the binder $(SET...: ...: ...)$.

(a) The set of real numbers between -10 and 10, but not equal to 0.

(b) The set of natural numbers which are not even.

(c) The set of all natural numbers which are multiples of 6.

(d) The set of natural numbers which are the sum of two squares of natural numbers.

Task 7. Calculate

(a) $\sum_{k=0}^{99} k$

(b) $\Pi_{k=0}^{99}(-1)^k$

(c) $(\text{SUM } k : -10 \leq k \leq 10 : k^{33})$

(d) $(\#k : -10 \leq k \leq 10 : k^2 > 9)$