Formale Systeme PS

Exercises, Week 5

Task 1. Show with calculations that $0 < x^2 - 2x + 1 < 9$ is equivalent to

$$x \neq 1 \wedge -2 < x < 4.$$

Task 2. Show with a calculation that the following formulas are tautologies

(a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

(b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 3. Check for every pair of propositions given below whether they are comparable (one is stronger than the other), or whether they are incomparable.

(a) $P \vee Q$ and $P \wedge Q$

(b) P and $\neg(P \vee Q)$

(c) P and $\neg(P \Rightarrow Q)$

Task 4. Are the following statements valid? Why?

(a) If $P \overset{val}{\models} Q$ and $Q \overset{val}{\models} R$ and $R \overset{val}{\models} S$, then $P \overset{val}{\models} S$.

(b) If $P \overset{val}{\models} Q$ and $P \overset{val}{\models} R$, then $Q \overset{val}{=} R$.

(c) If $P \overset{val}{\models} Q$ and $P \overset{val}{\models} R$, then Q and R are incomparable.

Task 5. Show with a calculation:

(a) $P \Rightarrow Q \overset{val}{\models} (P \wedge R) \Rightarrow (Q \wedge R)$

(b) $\neg(P \Rightarrow \neg Q)) \overset{val}{\models} (P \vee R) \wedge Q$

Task 6. Show with calculations that the following formulas are tautologies:

(a) $(P \wedge \neg P) \Rightarrow Q$

(b) $(P \Rightarrow (Q \wedge R)) \Rightarrow (P \Rightarrow (Q \vee R))$

(c) $(R \wedge (P \Rightarrow Q)) \Rightarrow ((R \Rightarrow P) \Rightarrow Q)$

Task 7. Show the correctness of the Monotonicity rules.