Formale Systeme PS

Exercises, Week 3

Task 1. Give the truth-tables of the abstract propositions (from Task 5. from last week):

(a) $(a \Rightarrow (b \Rightarrow a))$

(b) $((\neg(a \Rightarrow b)) \Leftrightarrow (a \wedge (\neg b)))$

(c) $((\neg(\neg a)) \Rightarrow ((\neg a) \wedge b))$

(d) $(a \Rightarrow ((b \wedge a) \vee c))$.

Task 2. For which values of a, b, and c one gets 0 in the truth-table of

$$(a \wedge (b \Rightarrow c)) \Rightarrow ((b \Rightarrow a) \wedge c) \ ?$$

Task 3. Try to find a canonical way of describing an abstract proposition that has a given truth-function. As guiding examples use the function

$$f = \begin{cases} \ a \ \ b & \\ (0,0) & \mapsto \quad 1 \\ (0,1) & \mapsto \quad 1 \\ (1,0) & \mapsto \quad 0 \\ (1,1) & \mapsto \quad 1. \end{cases}$$

whose corresponding abstract proposition is $((\neg a \wedge \neg b) \vee (\neg a \wedge b)) \vee (a \wedge b)$ and the function

$$g = \begin{cases} \ b \ \ c & \\ (0,0) & \mapsto \quad 1 \\ (0,1) & \mapsto \quad 0 \\ (1,0) & \mapsto \quad 1 \\ (1,1) & \mapsto \quad 0. \end{cases}$$

whose corresponding abstract proposition is $(\neg b \wedge \neg c) \vee (b \wedge \neg c)$.

Check first that both examples 'make sense'. Note that the principle works (except when the result is always 0), but the drawback is that one gets complex (long) formulas.

Task 4. Which Boolean functions correspond to:

(a) $a \Rightarrow \neg b$

(b) $\neg(a \wedge b)$

(c) $a \wedge (b \vee c)$.

(d) $\neg a \vee (a \wedge \neg c)$.

Which of these abstract propositions are equivalent?

Task 5. Show the equivalence of

(a) $\neg(b \vee \neg c)$ and $\neg b \wedge c$

(b) $\neg(a \Leftrightarrow b)$ and $\neg a \Leftrightarrow b$

(c) $(a \vee b) \wedge a$ and a

(d) $(a \vee b) \wedge b$ and $(b \wedge c) \vee (b \wedge \neg c)$.

Task 6.

(a) Show that there are infinitely many classes of equivalent propositions.

(b) Show that every class of equivalent propositions has infinitely many propositions.

Task 7. Give an example of a tautology with only one proposition variable a, without T or F, and with only parentheses and

(a) connective $\Rightarrow$

(b) connectives $\vee$ and $\neg$

(c) connectives $\wedge$ and $\neg$

(d) connective $\Leftrightarrow$.

Task 8. Show the equivalence of

(a) $F \Rightarrow a$ and T

(b) $T \Rightarrow a$ and a

(c) $a \Rightarrow F$ and $\neg a$

(d) $a \Rightarrow T$ and T.

Task 9. Give an example of two tautologies with two proposition variables a and b.