

Formale Systeme PS

Exercises, Week 2

Task 1. Check in each of the following cases whether the given rule is correct. If it is, give arguments to show this. If it is not, give a counter example.

- (a) There are K's which are also M's
All K's are L's

There are L's which are M's

- (b) No one K is an M
All K's are L's

No one L is an M

Task 2. Check in each of the following cases whether the given expressions are propositions or not. If yes, say whether this proposition is simple or complex. If no, give reasons why not.

- (a) It is freezing and yet it is not cold, because it is not windy.
- (b) Elephants don't exist.
- (c) Romeo and Juliet.
- (d) The capital of Austria is Salzburg.
- (e) Either $x < 0$ or $x = 0$ and $y < 0$.
- (f) If $x = 0$, then x .

Task 3. For each of the following concrete propositions, write an abstract proposition which corresponds to it:

- (a) I love you and will always be true to you.
- (b) If it is raining, then I will stay home and watch a movie.
- (c) $x^2 > 4$ if, and only if, $x > 2$ or $x < -2$.

(d) I will go to play tennis if you bring the balls with you.

Task 4. Give the following propositions in words again, with ‘it is raining’ for a , ‘it is windy’ for b , and ‘I am wet’ for c .

(a) $a \wedge \neg b$

(b) $\neg(a \vee b)$

(c) $(a \Rightarrow c) \vee (b \Rightarrow \neg a)$.

(d) $\neg\neg a$.

Task 5. Show how the following abstract propositions are built (Def. 2.3.1).

(a) $(a \Rightarrow (b \Rightarrow a))$

(b) $((\neg(a \Rightarrow b)) \Leftrightarrow (a \wedge (\neg b)))$

(c) $((\neg(\neg a)) \Rightarrow ((\neg a) \wedge b))$

(d) $(a \Rightarrow ((b \wedge a) \vee c))$.

Task 6. Show that:

- (a) In every abstract proposition in which the symbol \neg does not appear, the number of (occurrences of) proposition variables is one more than the number of connectives.
- (b) In every abstract proposition where no parenthesis are dropped out, the number of parenthesis is equal to twice the number of connectives.

Task 7.

- (a) Draw the trees of the abstract propositions of Exercise 5.
- (b) Give the main symbol for each of the abstract propositions of Exercise 5.

Task 8. Give all the sub-formulas of the abstract proposition

$$((a \wedge (\neg b)) \Rightarrow ((\neg c) \Rightarrow d)).$$

Task 9. Drop as many parenthesis as possible from the abstract propositions of Exercise 5.