Introduction to Concurrency and Verification

Exam, January 29, 2008

Task 1. (4*5) For each of the following pairs of CCS expressions decide if they are strongly bisimilar. If they are, then prove it (by providing a bisimulation, for example). If they are not, then prove it by providing a distinguishing formula in Hennessy-Milner logic or a universal winning strategy for the attacker in a bisimilarity game.

(1) $b.a.0 + b.0$ and $b.(a.0 + b.0)$;

(2) $a.0 \mid b.0$ and $a.b.0 + b.a.0$;

(3) $(a.0 \mid b.0) + c.a.0$ and $a.0 \mid (b.0 + c.0)$;

(4) $(a.0 \mid b.0) \mid (c.a.0)$ and $(a.b.0 + b.a.0) \mid (c.a.0)$.

Task 2. (20) Is it true that

$$(P + Q) \mid R \sim P \mid R + Q \mid R$$

for any CCS processes P, Q, and R? If yes, prove your claim by providing a bisimulation. If not, give a counter example.

Task 3. (6+7+7) A binary relation R, $R \subseteq S \times S$, over the set of states of an LTS $(S, Act, (\xrightarrow{a} \mid a \in Act))$ is a *ready simulation* if and only if whenever $(s, t) \in R$ and $a \in Act$ we have:

- if $s \xrightarrow{a} s'$, then there exists a state t' such that $t \xrightarrow{a} t'$ and $(s', t') \in R$, and
- if $t \xrightarrow{a}$, then $s \xrightarrow{a}$.

We say that t *ready simulates* s, notation $s \sqsubseteq_{RS} t$ if and only if there is a ready simulation R with $(s, t) \in R$. Two states s and t are *ready simulation equivalent*, notation $s \equiv_{RS} t$ if $s \sqsubseteq_{RS} t$ and $t \sqsubseteq_{RS} s$.

(1) Do the following relations hold?

$$a.0 \sqsubseteq_{RS} a.a.0, \quad a.b.0 + a.c.0 \sqsubseteq_{RS} a.(b.0 + c.0)$$

(2) Show that strong bisimilarity is stronger than ready simulation equivalence, that is $s \sim t \implies s \equiv_{RS} t$.

(3) Is there a CCS process that can ready simulate any other CCS process? Argument your answer.

Task 4. (10+10) Consider the LTS

$$\begin{tikzcd}
s_0 \arrow[r, bend left, "a"] & s_1 \arrow[l, bend left, "a"] \arrow[r, bend left, "a"] & s_2 \arrow[l, bend left, "a"] \arrow[d, "c"] \\
s_3 \arrow[u, "b"] \arrow[rr, "b"] & & s_4 \arrow[loop right, "c"]
\end{tikzcd}$$

Determine the set of states for which

(1) $\phi \, \mathcal{U}^s \psi$ holds,

(2) $\phi \, \mathcal{U}^w \psi$ holds.

Here $\mathcal{U}^s$ and $\mathcal{U}^w$ denote the strong and the weak until operator defined as

$$\phi \, \mathcal{U}^s \psi \overset{\min}{=} \psi \vee (\phi \ \wedge \langle Act \rangle tt \wedge [Act](\phi \, \mathcal{U}^s \psi))$$

and

$$\phi \, \mathcal{U}^w \psi \overset{\max}{=} \psi \vee (\phi \ \wedge [Act](\phi \, \mathcal{U}^w \psi)).$$

For the formulas $\phi = \langle a \rangle tt$ and $\psi = \langle c \rangle tt$.

Task 5. (10+10) Show that, for all CCS processes P and Q:

(1) If $P \overset{\tau}{\Longrightarrow} P'$, then $P \mid Q \overset{\tau}{\Longrightarrow} P' \mid Q$.
(Hint: Use induction on the number of τ-steps in the transition $P \overset{\tau}{\Longrightarrow} P'$.)

(2) If $P \overset{a}{\Longrightarrow} P'$ and $Q \overset{\bar{a}}{\longrightarrow} Q'$, then $P \mid Q \overset{\tau}{\Longrightarrow} P' \mid Q'$.
(Hint: Use induction on the number of τ-steps in the transition $P \overset{a}{\Longrightarrow} P'$ and/or (1).)