Temporal Logics

LTL

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Turing Award 1996

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Verification

$$M, i \overset{?}{\models} \varphi$$

Given a model
(Kripke structure)
with initial state

Given a property
in temporal logic

Check if the model
satisfies the property

Synthesis

$$M, i \overset{!}{\models} \varphi$$

Given a property in temporal logic

Construct a model that satisfies the property

Temporal logics

express
properties of worlds
that change
over time

without explicitly
referring to time:
eventually, next time, globally,…

Examples:

Nothing bad will ever happen.

Something good will eventually happen.

LTL

Linear Temporal Logic

expresses properties
over a single path

LTL

Linear Temporal Logic

expresses properties over a single path

$p \in AP$

- Atomic propositions
- Boolean connectives
- Temporal operators
- Path quantifiers

$\neg$, $\vee$,
$\wedge$, $\Rightarrow$, $\Leftrightarrow$

X (next time), U (until),
F (future), G (globally), R (releases)

A (for all) — implicit

LTL syntax

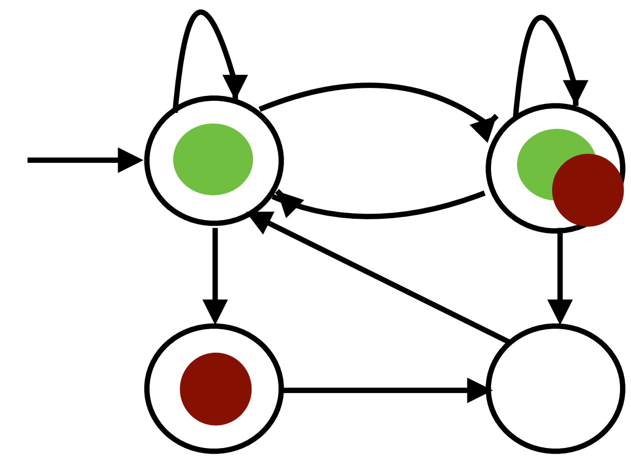
- If φ is a path formula, then $A\varphi$ is a state formula.
- If φ and ψ are path formulas, then so are

$$\neg\varphi,\ \varphi \vee \psi,\ p \in AP,\ X\psi,\ \varphi\, U\, \psi$$

$$\varphi \wedge \psi,\ \varphi \Rightarrow \psi,\ \varphi \Leftrightarrow \psi$$

$$F\varphi,\ G\varphi,\ \varphi\, R\, \psi$$

LTL semantics



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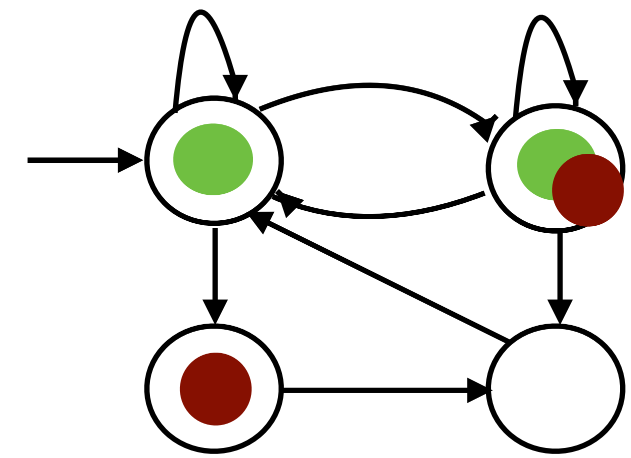
wrt. a fixed Kripke structure



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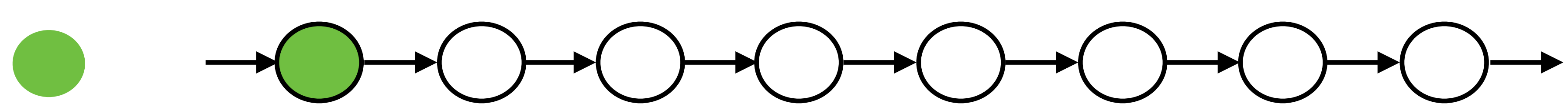

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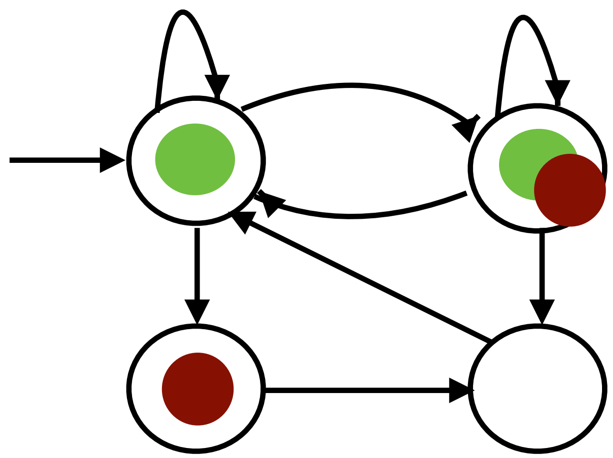

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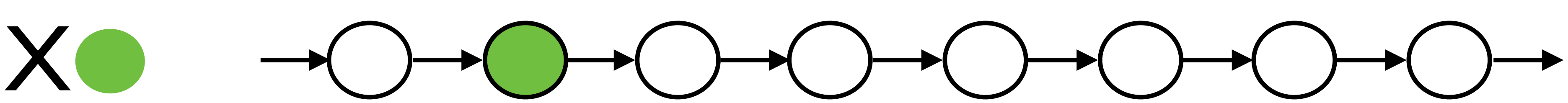

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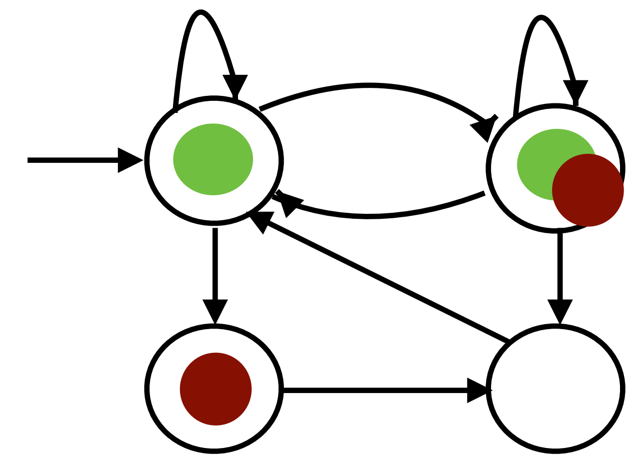

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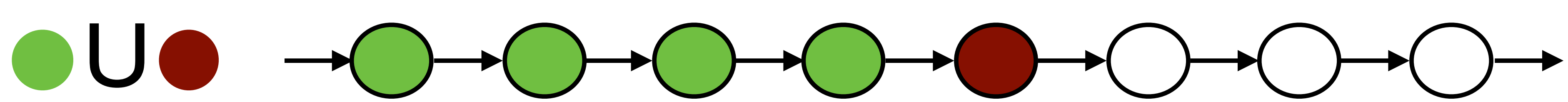

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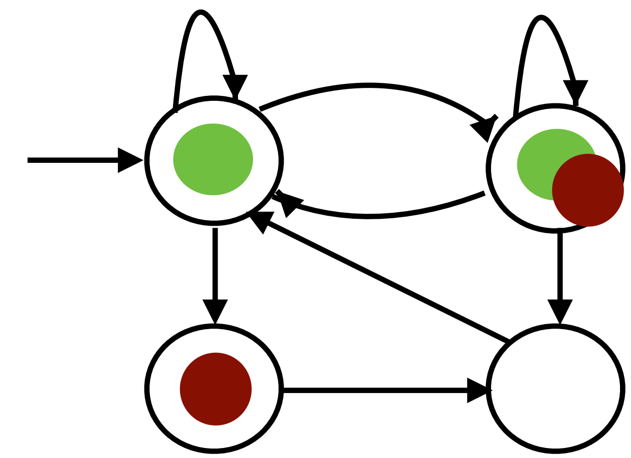
LTL semantics



$$F\varphi = T\,U\,\varphi$$
$$T = p \vee \neg p$$

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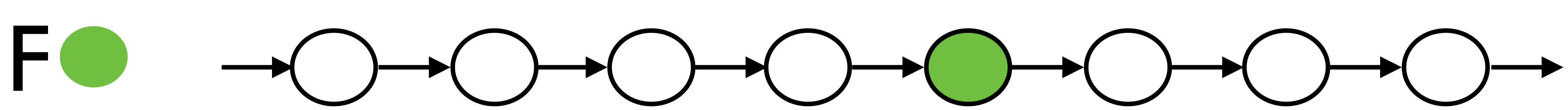
$$F\varphi,\ G\varphi,\ \varphi\,R\,\psi$$



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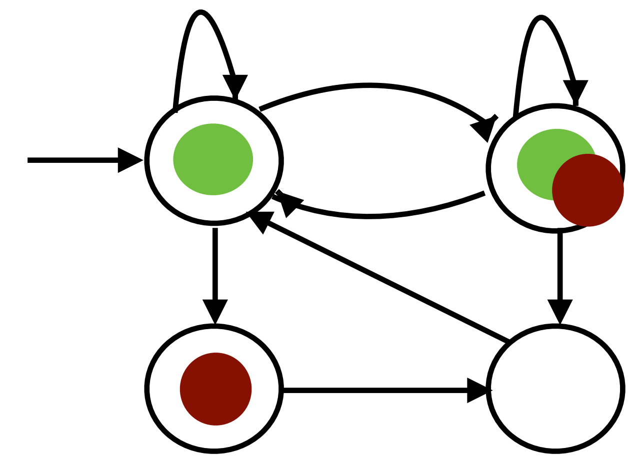
LTL semantics



$G\varphi = \neg F \neg \varphi$

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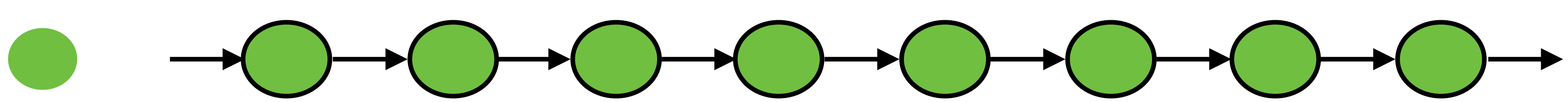
G



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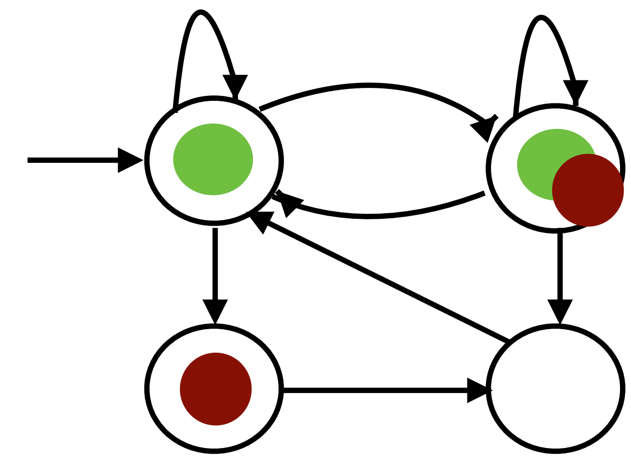
LTL semantics



$\varphi \, R \, \psi = \, ?$

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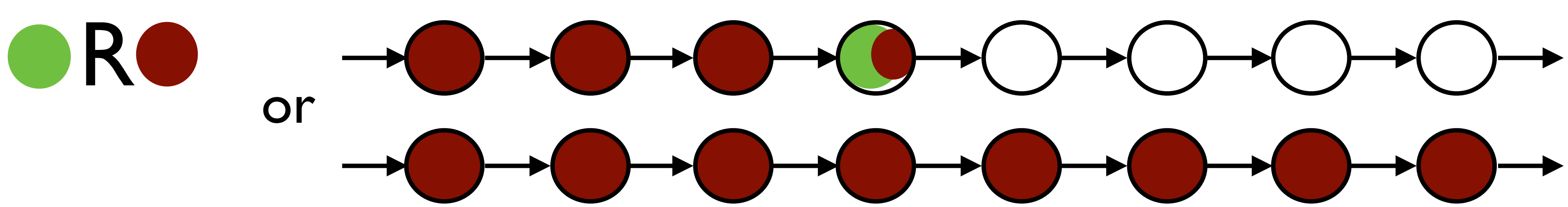

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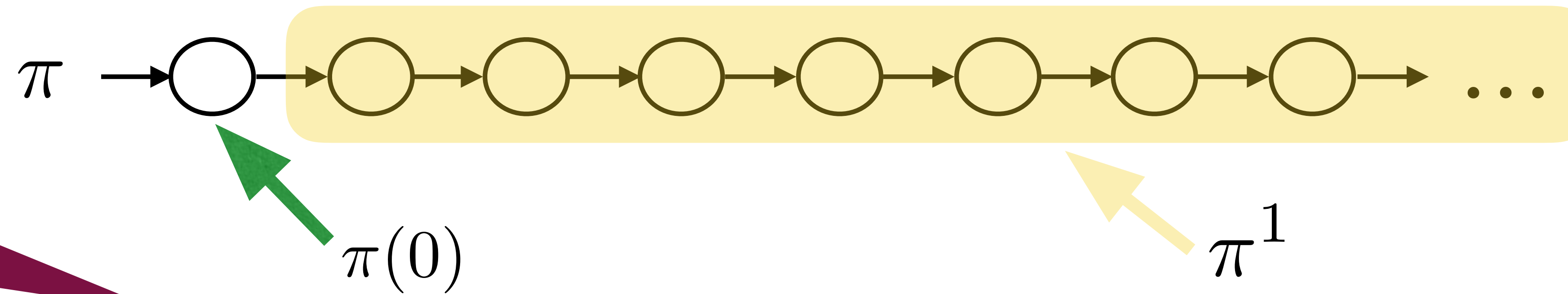


LTL semantics

$$\pi \models p \quad \text{iff} \quad \pi(0) \text{ is labelled by } p$$

$$\pi \models \neg\varphi \quad \text{iff} \quad \pi \not\models \varphi$$

$$\pi \models \varphi \vee \psi \quad \text{iff} \quad \pi \models \varphi \text{ or } \pi \models \psi$$

$$\pi \models X\varphi \quad \text{iff} \quad \pi^1 \models \varphi$$

$$\pi \models \varphi U \psi \quad \text{iff} \quad \exists i \geqslant 0.\ \pi^i \models \psi \wedge \forall j < i.\ \pi^j \models \varphi$$

$$\pi \models F\varphi \quad \text{iff} \quad \exists i \geqslant 0.\ \pi^i \models \varphi$$

$$\pi \models G\varphi \quad \text{iff} \quad \forall i \geqslant 0.\ \pi^i \models \varphi$$

$$\pi \models \varphi R \psi \quad \text{iff} \quad \forall i \geqslant 0. (\forall j < i.\ \pi^j \not\models \varphi \Rightarrow \pi^i \models \psi)$$

Homework task

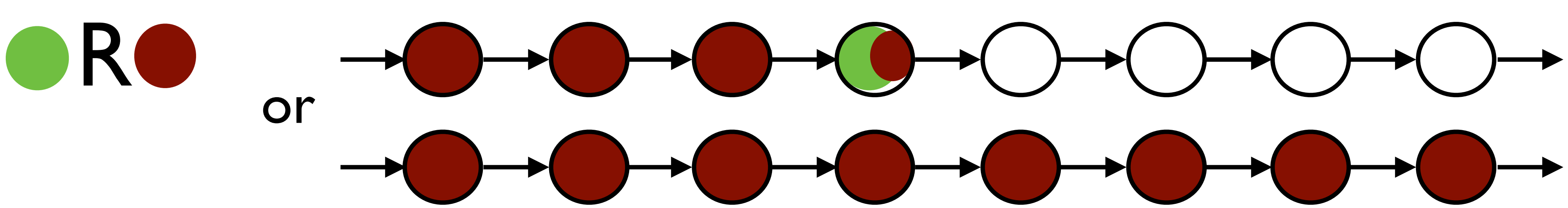
Prove that

$$\pi \models \varphi \, R \, \psi \qquad \text{iff} \qquad \forall i \geqslant 0. \, (\forall j < i. \, \pi^j \not\models \varphi \Rightarrow \pi^i \models \psi)$$

relating the formula above with the derived meaning

$$\varphi \, R \, \psi = (\psi \; U \; (\varphi \wedge \psi)) \vee G\psi$$

from the informal intended semantics



LTL examples

Liveness

request ⇒ F grant — A request will eventually be granted.

🌧 ⇒ F ☀ — After the rain, the sun will shine.

FG ☀ — Eventually, there will be only sunshine.

GF ☀ — Infinitely often there will be sunshine.

G¬🌧 — No rain ever.

Safety

From every state a ☀ state is reachable ?

not expressible in LTL,
expressible in CTL