Temporal Logics

LTL

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Temporal Logics

LTL

Amir Pnueli 1977
Turing Award 1996

Ana Sokolova UNIVERSITY of SALZBURG

Verification

$$M, i \overset{?}{\models} \varphi$$

Verification

Given a model
(Kripke structure)
with initial state

$$M, i \overset{?}{\models} \varphi$$

Given a property
in temporal logic

Verification

$$M, i \overset{?}{\models} \varphi$$

Given a model (Kripke structure) with initial state

Given a property in temporal logic

Check if the model satisfies the property

Synthesis

$$M, i \overset{!}{\models} \varphi$$

Given a property in temporal logic

Synthesis

$$M, i \overset{!}{\models} \varphi$$

Given a property in temporal logic

Construct a model that satisfies the property

Temporal logics

express
properties of worlds
that change
over time

without explicitly
referring to time:
eventually, next time, globally,…

Temporal logics

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eventually, next time, globally,. . .

Examples:

Nothing bad will ever happen.

Something good will eventually happen.

LTL

Linear Temporal Logic

LTL

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expresses properties
over a single path

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Linear Temporal Logic

expresses properties over a single path

- Atomic propositions
- Boolean connectives
- Temporal operators
- Path quantifiers

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$p \in AP$

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$\neg$, $\vee$,
$\wedge$, $\Rightarrow$, $\Leftrightarrow$

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A (for all) — implicit

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Linear Temporal Logic

expresses properties over a single path

$p \in AP$

- Atomic propositions
- Boolean connectives
- Temporal operators
- Path quantifiers

$\neg$, $\vee$,
$\wedge$, $\Rightarrow$, $\Leftrightarrow$

X (next time), U (until),
F (future), G (globally), R (releases)

A (for all) — implicit

LTL syntax

- If φ is a path formula, then $A\varphi$ is a state formula.

- If φ and ψ are path formulas, then so are

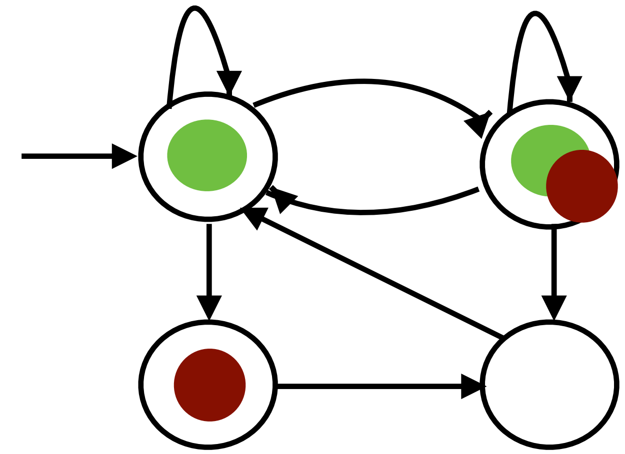
$$\neg\varphi,\ \varphi \vee \psi,\ p \in AP,\ X\psi,\ \varphi\, U\, \psi$$

$$\varphi \wedge \psi,\ \varphi \Rightarrow \psi,\ \varphi \Leftrightarrow \psi$$

$$F\varphi,\ G\varphi,\ \varphi\, R\, \psi$$

wrt. a fixed Kripke structure

LTL semantics



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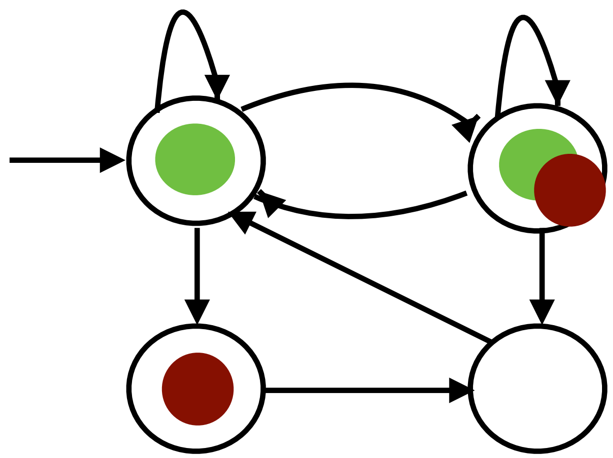

on all paths starting in the state φ holds

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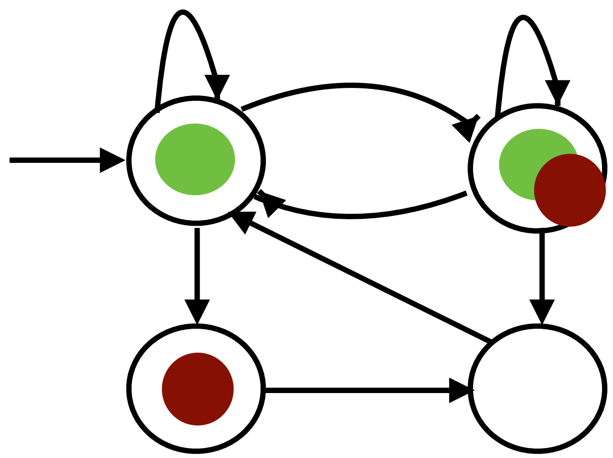

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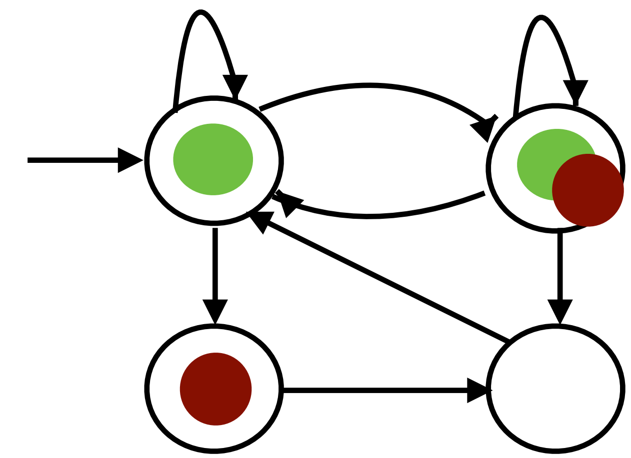

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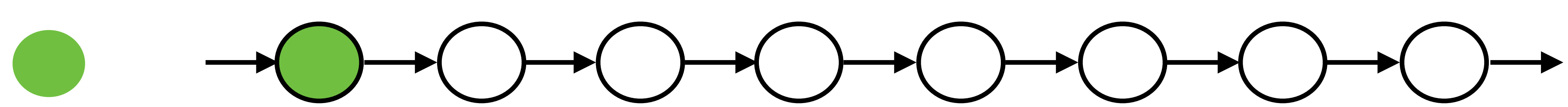

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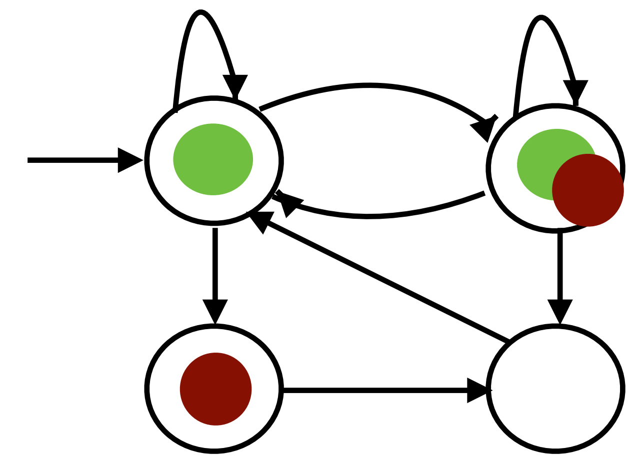

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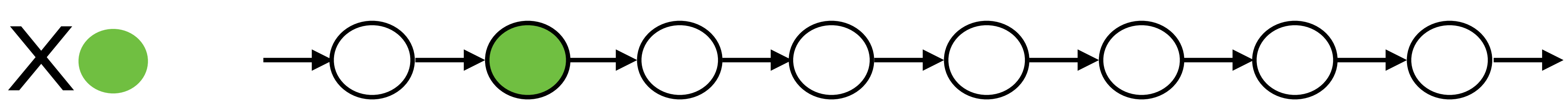

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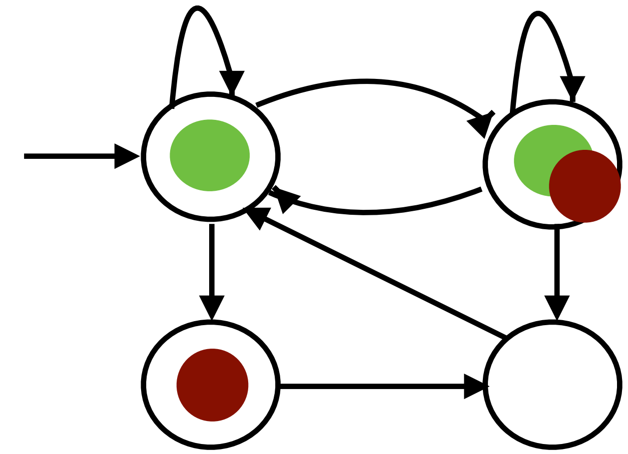

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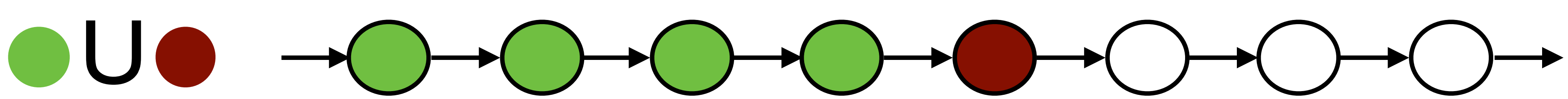
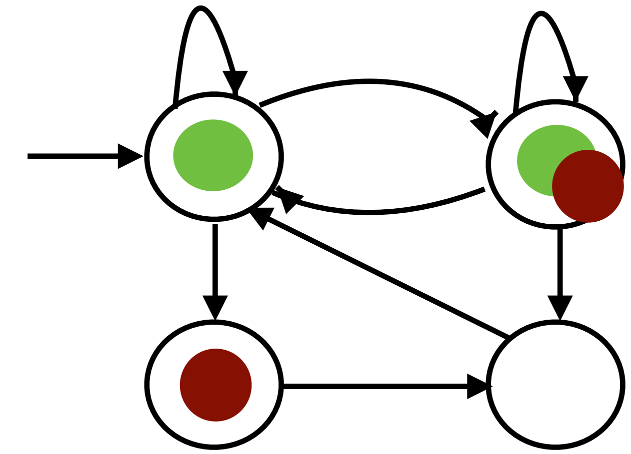


wrt. a fixed Kripke structure

LTL semantics



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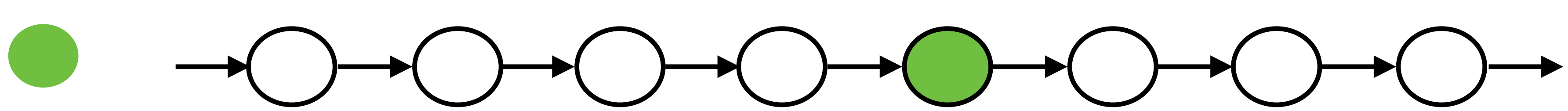
F 

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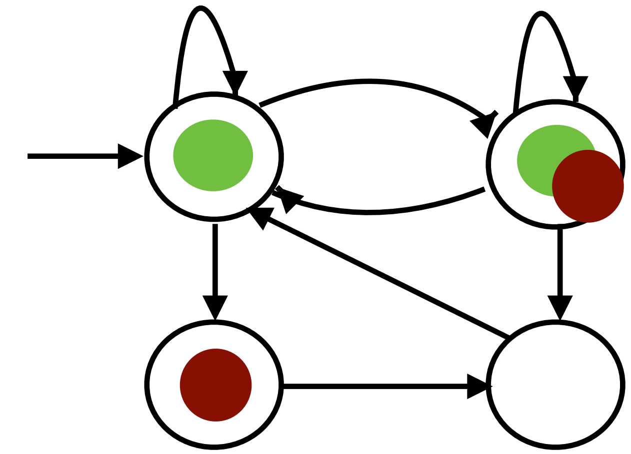
LTL semantics



$$F\varphi = T\,U\,\varphi$$
$$T = p \lor \neg p$$

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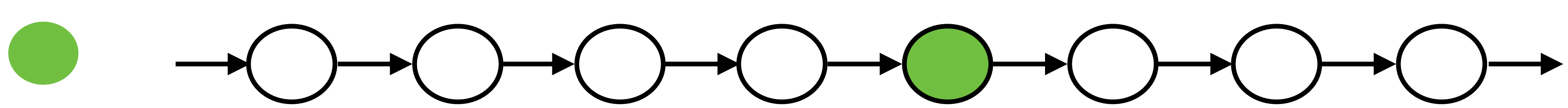
F 

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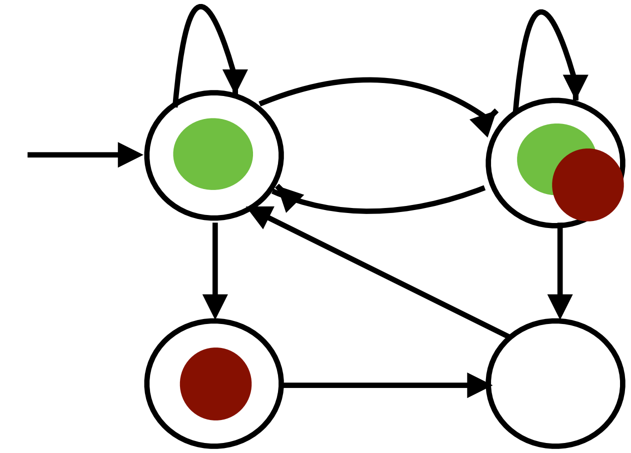

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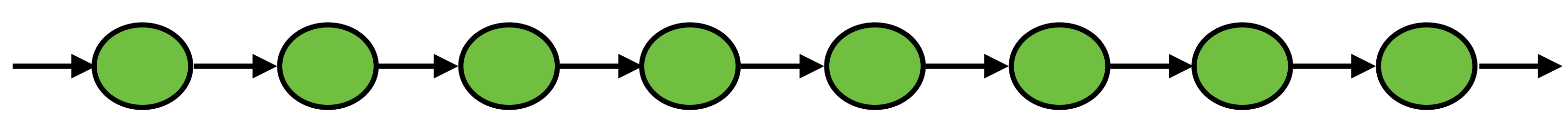
G



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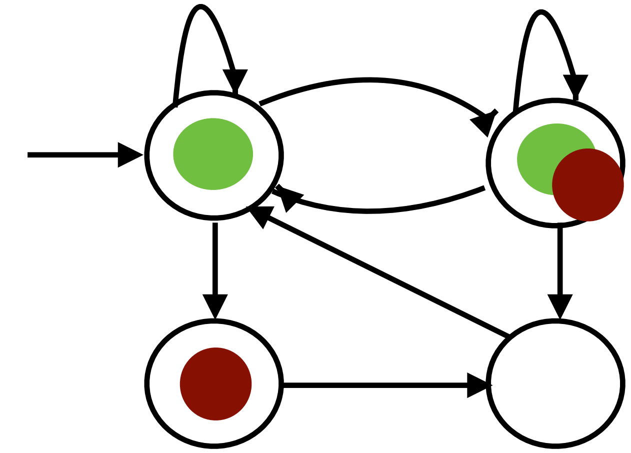

$$G\varphi = \neg F \neg \varphi$$

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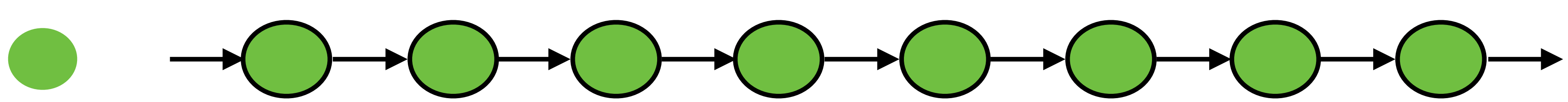
G 

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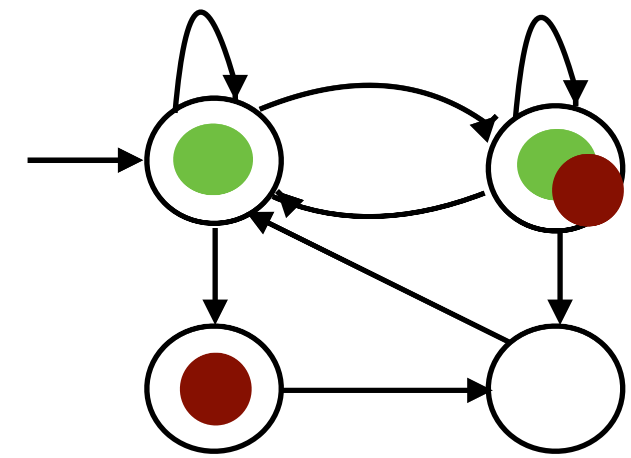

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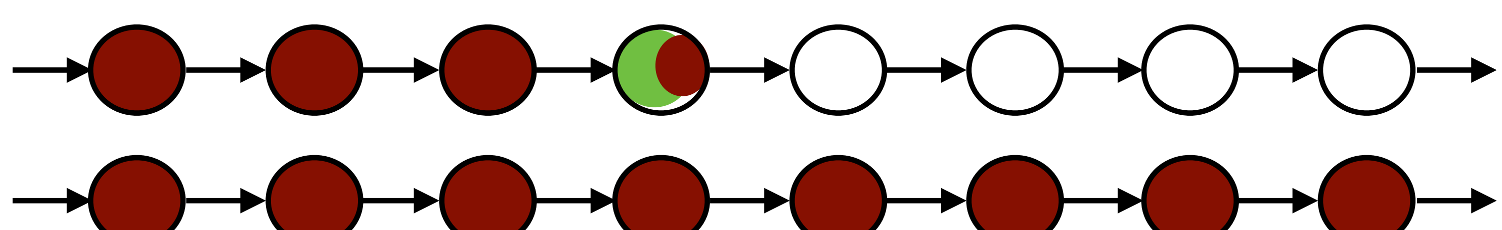

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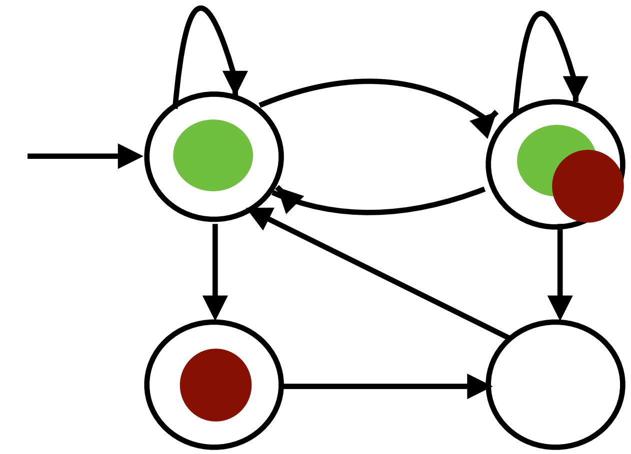

$$\varphi\, R\, \psi = ?$$

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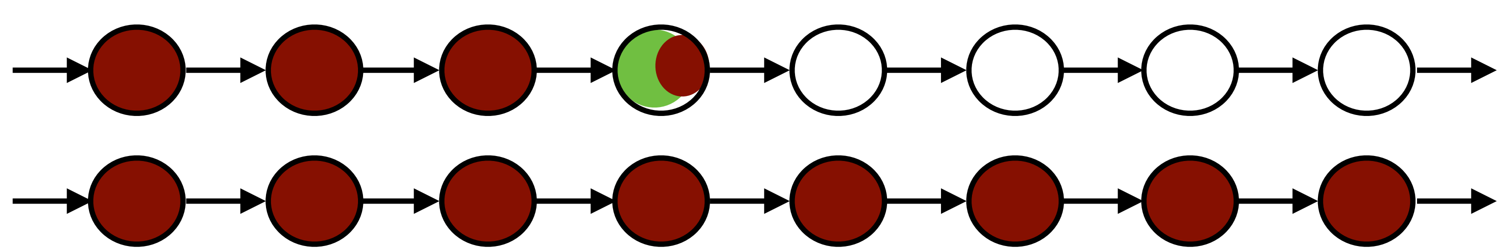
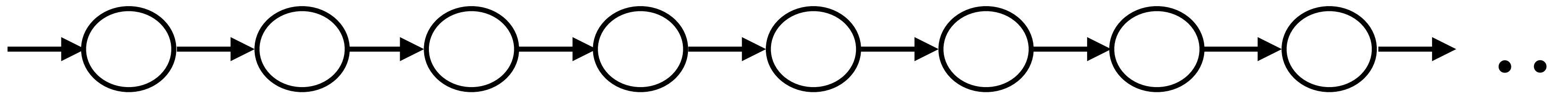


wrt. a fixed Kripke structure

LTL semantics

wrt. a fixed Kripke structure

π



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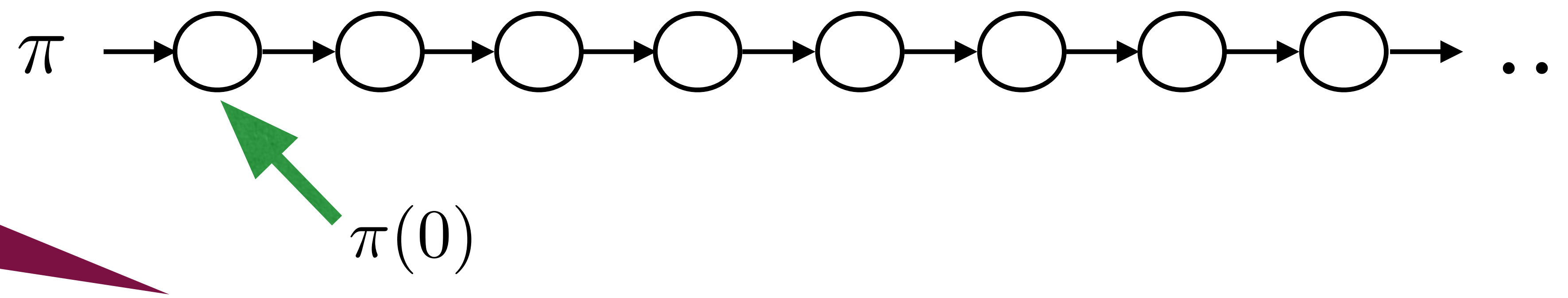

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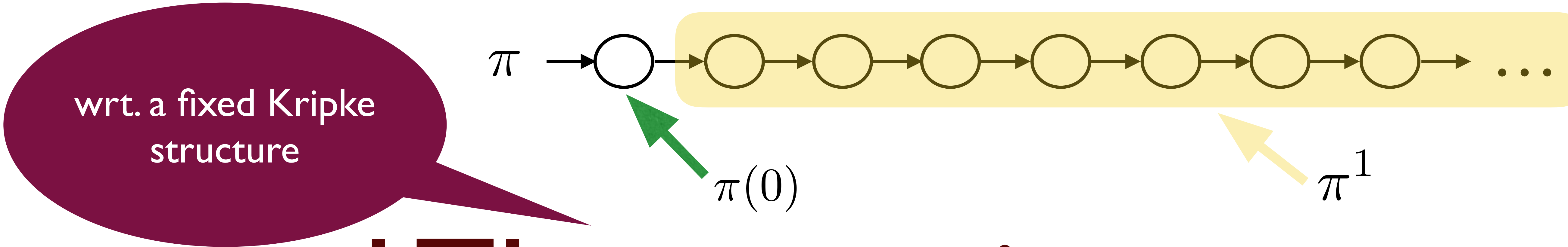

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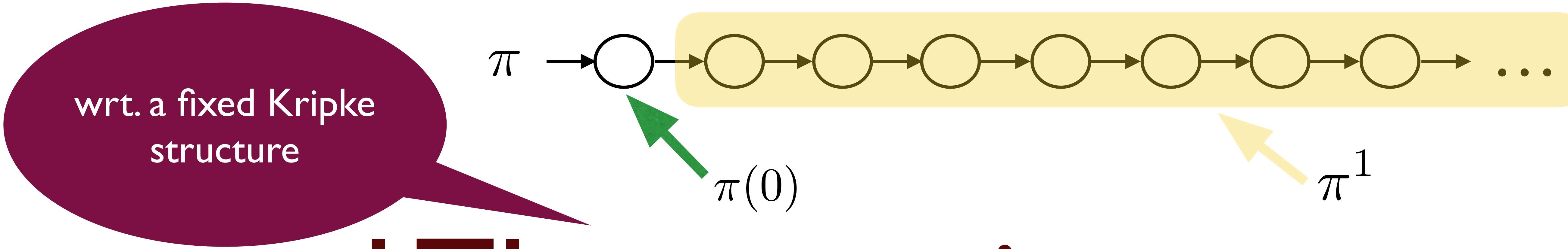

LTL semantics

$\pi \models p$	iff	$\pi(0)$ is labelled by p
$\pi \models \neg\varphi$	iff	$\pi \not\models \varphi$
$\pi \models \varphi \vee \psi$	iff	$\pi \models \varphi$ or $\pi \models \psi$
$\pi \models X\varphi$	iff	$\pi^1 \models \varphi$
$\pi \models \varphi U \psi$	iff	$\exists i \geqslant 0.\ \pi^i \models \psi \wedge \forall j < i.\ \pi^j \models \varphi$

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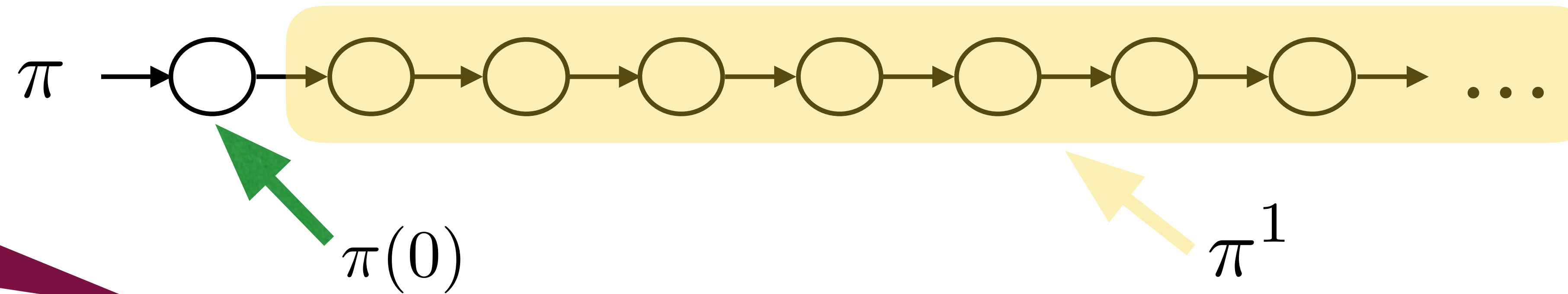

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$\pi \models X\varphi$	iff	$\pi^1 \models \varphi$
$\pi \models \varphi U \psi$	iff	$\exists i \geqslant 0.\ \pi^i \models \psi \wedge \forall j < i.\ \pi^j \models \varphi$
$\pi \models F\varphi$	iff	$\exists i \geqslant 0.\ \pi^i \models \varphi$
$\pi \models G\varphi$	iff	$\forall i \geqslant 0.\ \pi^i \models \varphi$
$\pi \models \varphi R \psi$	iff	$\forall i \geqslant 0.(\forall j < i.\ \pi^j \not\models \varphi \Rightarrow \pi^i \models \psi)$

Homework task

Prove that

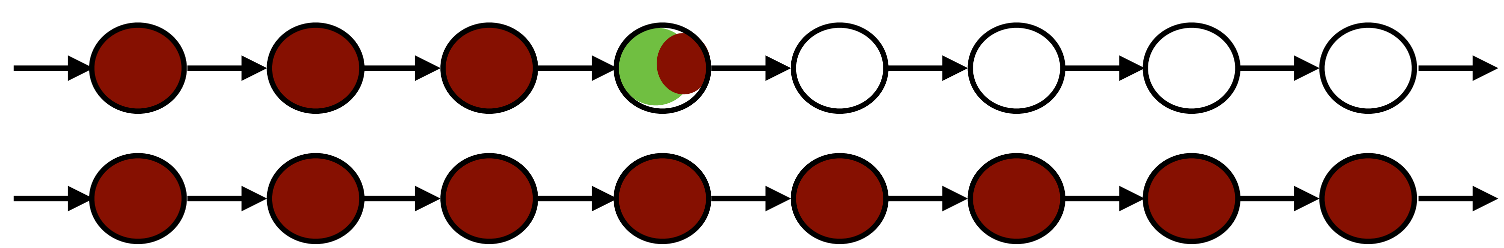
$$\pi \models \varphi \, R \, \psi \qquad \text{iff} \qquad \forall i \geqslant 0. \, (\forall j < i. \, \pi^j \not\models \varphi \Rightarrow \pi^i \models \psi)$$

relating the formula above with the derived meaning

$$\varphi \, R \, \psi = (\psi \; U \; (\varphi \wedge \psi)) \vee G\psi$$

from the informal intended semantics

or



LTL examples

request ⇒ F grant

🌧️ ⇒ F ☀️

FG ☀️

GF ☀️

G¬🌧️

LTL examples

request ⇒ F grant — A request will eventually be granted.

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LTL examples

request ⇒ F grant	A request will eventually be granted.
🌧 ⇒ F ☀️	After the rain, the sun will shine.
FG ☀️	
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G¬🌧	

LTL examples

request ⇒ F grant — A request will eventually be granted.

🌧 ⇒ F ☀️ — After the rain, the sun will shine.

FG ☀️ — Eventually, there will be only sunshine.

GF ☀️

G¬🌧

LTL examples

request $\Rightarrow$ F grant	A request will eventually be granted.
🌧️ $\Rightarrow$ F ☀️	After the rain, the sun will shine.
FG ☀️	Eventually, there will be only sunshine.
GF ☀️	Infinitely often there will be sunshine.
G¬🌧️	

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G¬🌧	No rain ever.

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Safety

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Safety

From every state a ☀️ state is reachable ?

LTL examples

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not expressible in LTL,
expressible in CTL