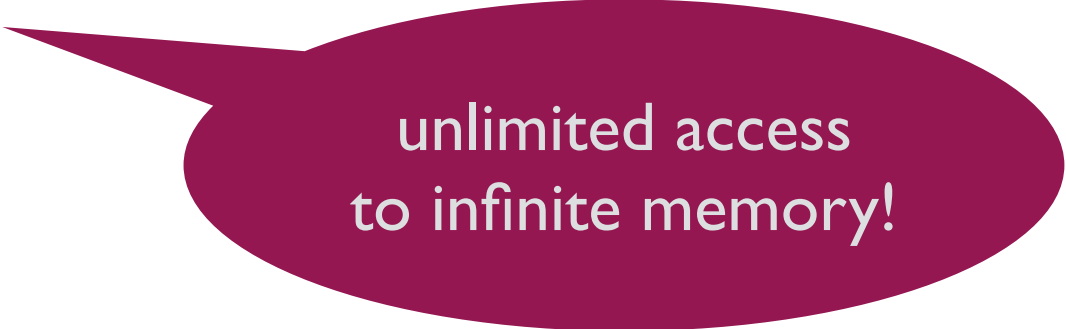


Turing Machines

= finite control (automaton states)
+ (potentially) infinite tape
+ head for reading/writing



unlimited access
to infinite memory!

Turing Machine

Definition

A Turing machine M is a tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, !, q_{rej})$ where

Q is a finite set of states

Σ is the input alphabet ($\square \notin \Sigma$)

Γ is the tape alphabet ($\Sigma \subseteq \Gamma, \square \in \Gamma$)

$\delta: Q \setminus \{!, q_{rej}\} \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$ is the transition function

q_0 is the initial state, $q_0 \in Q$

$!$ (or q_{acc}) is the accept state, $! \in Q$

q_{rej} is the reject state, $q_{rej} \in Q$

or $\{L, R, N\}$

no transitions from
 $!$ and q_{rej}

$\delta(q, a) = (r, b, X)$ means that in a state q , reading the symbol a from the cell on which the head is positioned, the TM changes to state r , writes b in place of a , and moves the head for one cell in direction X

Turing machines

Compute via configurations

Given $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, q_{rej})$ a configuration of M is an element in

$Q \times \Gamma^* \times \mathbb{N}$ (current state, input word, position of the head)
an initial configuration is $(q_0, w, 1)$.

(q, w, i)
shortly uqv
if $w = uv, |u| = i-1$

one-step computation

$uaqbv \vDash uracv$ iff $\delta(q, b) = (r, c, L)$
 $uaqbv \vDash uacrv$ iff $\delta(q, b) = (r, c, R)$
 $qbv \vDash rcv$ iff $\delta(q, b) = (r, c, L)$
 $uaq \vDash uacr$ iff $\delta(q, \square) = (r, c, R)$

Definition (Turing recognisable language)

The language recognised / accepted by a Turing machine
 $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, q_{rej})$ is

$$L(M) = \{w \in \Sigma^* \mid (q_0, w, 1) \vDash^* (\sqcup, u, j) \text{ for some } u \text{ and } j\}$$

zero-or-more-steps
computation

Turing machines

On input w may

- (1) **accept** w , if $(q_0, w, l) \models^* (!, u, j)$ for some u and j
- (2) **reject** w , if $(q_0, w, l) \models^* (q_{rej}, u, j)$ for some u and j
- (3) **loop**, i.e., never reach $!$ or q_{rej}

both (2) and (3)
are
not accepting

A decider TM on input w may only

- (1) **accept** w , if $(q_0, w, l) \models^* (!, u, j)$ for some u and j
- (2) **reject** w , if $(q_0, w, l) \models^* (q_{rej}, u, j)$ for some u and j

deciders
do not
loop

Definition (Turing decidable language)

A language L is Turing decidable if $L = L(M)$ for some Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, !, q_{rej})$ which is a decider.