# Nonregular languages

every long enough word of a regular language can be pumped

## Theorem (Pumping Lemma)

If L is a regular language, then there is a number $p \in \mathbb{N}$ (the pumping length) such that for any $w \in L$ with $|w| \geq p$, there exist $x, y, z \in \Sigma^*$ such that $w = xyz$ and

1. $xy^iz \in L$, for all $i \in \mathbb{N}$
2. $|y| > 0$
3. $|xy| \leq p$

Proof sketch easy, using the pigeonhole principle

## Example "corollary"

$L = \{ 0^n 1^n \mid n \in \mathbb{N}\}$ is nonregular.

Note the logical structure!

# Context-free Grammars
# and
# Push-down Automata

# Context-free Grammars

## Informal example

$\Sigma = \{0,1\}$

$G_1$: $S \to 0S1, S \to A, A \to \varepsilon$

alphabet (terminals)

S, A are variables (nonterminals)

S is initial nonterminal

production rules "context-free!"

Generates the language $L(G_1) = \{0^n 1^n \mid n \in \mathbb{N}\}$

context-free language

# CFG

## Definition

A context-free grammar G is a tuple $G = (V, \Sigma, R, S)$ where

V is a finite set of variables (nonterminal symbols, nonterminals)
$\Sigma$ is a finite alphabet (of terminal symbols, terminals)
R is a finite set of (production) rules, $R \subseteq V \times (\Sigma \cup V)^*$
S is the initial nonterminal, $S \in V$

## In the example G

$G_1 = (V, \Sigma, R, S)$ for

$V = \{S, A\}$

$\Sigma = \{0, 1\}$

$R = \{ (S, 0S1), (S, A), (A, \varepsilon)\}$

# CFG

u derives v

context-free language

## Derivations

uAv yields uwv

Given $G = (V, \Sigma, R, S)$ we have $uAv \Rightarrow uwv$ for $u, v, w \in (\Sigma \cup V)^*$, $A \rightarrow w \in R$

and $u \Rightarrow^* v$ if $u = v$

or there exists a sequence $u_1, u_2, .., u_k$ for $k \geq 0$ such that
$u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow .. \Rightarrow u_k \Rightarrow v$

## Definition

The language generated by a context-free grammar $G = (V, \Sigma, R, S)$ is

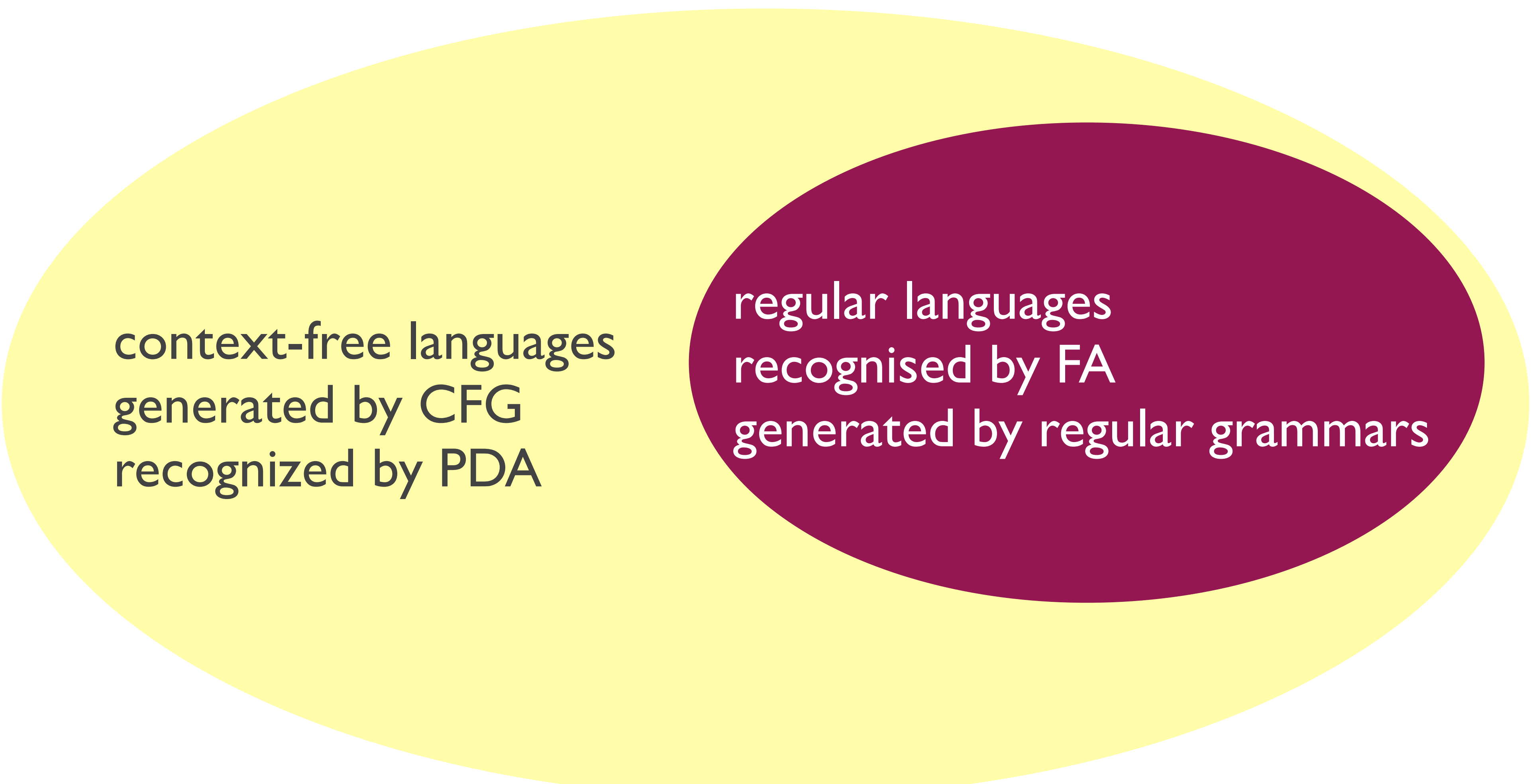
$$L(G) = \{w \in \Sigma^* | \ S \Rightarrow^* w\}$$

all words consisting only of terminals, that can be derived from the initial nonterminal

# Regular vs. CF languages

**Theorem RL-CFL**

The class of regular languages is contained in the class of context -free languages.



# Non-context-free languages

every long enough word of a context-free language can be pumped at two places simultaneously

## Theorem (Pumping Lemma)

If L is a context-free language, then there is a number $p \in \mathbb{N}$ (the pumping length) such that for any $w \in L$ with $|w| \geq p$, there exist $u, v, x, y, z \in \Sigma^*$ such that $w = uvxyz$ and

1. $uv^ixy^iz \in L$ , for all $i \in \mathbb{N}$
2. $|vy| > 0$
3. $|vxy| \leq p$

Proof sketch easy, using the pigeonhole principle

## Example "corollary"

$L = \{ a^n b^n c^n \mid n \in \mathbb{N}\}$ is non-context-free.

Note the logical structure!

# Properties of CF languages

**Theorem CF1**

The class of regular languages is closed under union

but not under intersection!

**Theorem CF2**

The class of regular languages is closed under concatenation

and not under complement!

**Theorem CF3**

The class of regular languages is closed under Kleene star

**Theorem CF4**

The intersection of a regular language and a context-free language is context-free