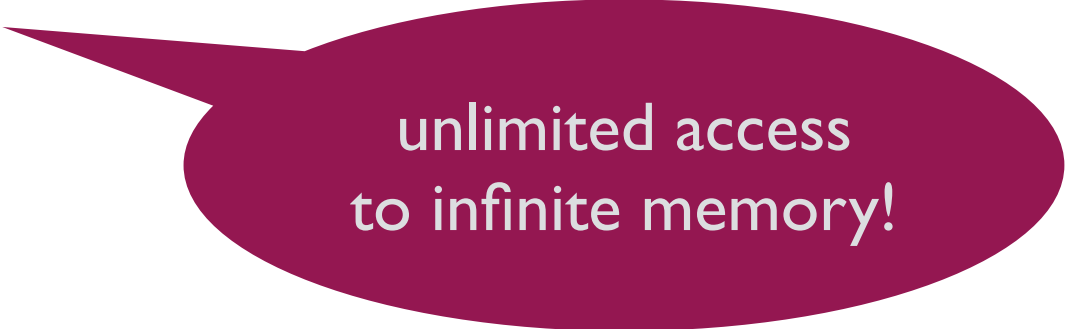


# Turing Machines

= finite control (automaton states)  
+ (potentially) infinite tape  
+ head for reading/writing



unlimited access  
to infinite memory!

# Turing Machine

## Definition

A Turing machine  $M$  is a tuple  $M = (Q, \Sigma, \Gamma, \delta, q_0, !, q_{rej})$  where

$Q$  is a finite set of states

$\Sigma$  is the input alphabet ( $\square \notin \Sigma$ )

$\Gamma$  is the tape alphabet ( $\Sigma \subseteq \Gamma, \square \in \Gamma$ )

$\delta: Q \setminus \{!, q_{rej}\} \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$  is the transition function

$q_0$  is the initial state,  $q_0 \in Q$

$!$  (or  $q_{acc}$ ) is the accept state,  $! \in Q$

$q_{rej}$  is the reject state,  $q_{rej} \in Q$

or  $\{L, R, N\}$

no transitions from  
 $!$  and  $q_{rej}$

$\delta(q, a) = (r, b, X)$  means that in a state  $q$ , reading the symbol  $a$  from the cell on which the head is positioned, the TM changes to state  $r$ , writes  $c$  in place of  $b$ , and moves the head for one cell in direction  $X$

# Turing machines

## Compute via configurations

Given  $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, q_{rej})$  a configuration of  $M$  is an element in

$Q \times \Gamma^* \times \mathbb{N}$  (current state, input word, position of the head)  
an initial configuration is  $(q_0, w, 1)$ .

$(q, w, i)$   
shortly  $uqv$   
if  $w = uv, |u| = i-1$

one-step computation

$uaqbv \vDash uracv$  iff  $\delta(q, b) = (r, c, L)$   
 $uaqbv \vDash uacrv$  iff  $\delta(q, b) = (r, c, R)$   
 $qbv \vDash rcv$  iff  $\delta(q, b) = (r, c, L)$   
 $uaq \vDash uacr$  iff  $\delta(q, \square) = (r, c, R)$

## Definition (Turing recognisable language)

The language recognised / accepted by a Turing machine  
 $M = (Q, \Sigma, \Gamma, \delta, q_0, \sqcup, q_{rej})$  is

$$L(M) = \{w \in \Sigma^* \mid (q_0, w, 1) \vDash^* (\sqcup, u, j) \text{ for some } u \text{ and } j\}$$

zero-or-more-steps  
computation

# Turing machines

## On input $w$ may

- (1) **accept**  $w$ , if  $(q_0, w, l) \models^* (!, u, j)$  for some  $u$  and  $j$
- (2) **reject**  $w$ , if  $(q_0, w, l) \models^* (q_{rej}, u, j)$  for some  $u$  and  $j$
- (3) **loop**, i.e., never reach  $!$  or  $q_{rej}$

both (2) and (3)  
are  
not accepting

## A decider TM on input $w$ may only

- (1) **accept**  $w$ , if  $(q_0, w, l) \models^* (!, u, j)$  for some  $u$  and  $j$
- (2) **reject**  $w$ , if  $(q_0, w, l) \models^* (q_{rej}, u, j)$  for some  $u$  and  $j$

deciders  
do not  
loop

## Definition (Turing decidable language)

A language  $L$  is Turing decidable if  $L = L(M)$  for some Turing machine  $M = (Q, \Sigma, \Gamma, \delta, q_0, !, q_{rej})$  which is a decider.