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$L = \{ 0^n 1^n \mid n \in \mathbb{N}\}$ is nonregular.

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Context-free Grammars
and
Push-down Automata

Context-free Grammars

Informal example

$\Sigma = \{0,1\}$

G_1: $S \rightarrow 0S1, S \rightarrow A, A \rightarrow \varepsilon$

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alphabet (terminals)

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context-free language

CFG

Definition

A context-free grammar G is a tuple G = (V, ∑, R, S) where

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CFG

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and $u \Rightarrow^* v$ if $u = v$

or there exists a sequence $u_1, u_2, .., u_k$ for $k \geq 0$ such that

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Theorem RL-CFL

The class of regular languages is contained in the class of context -free languages.

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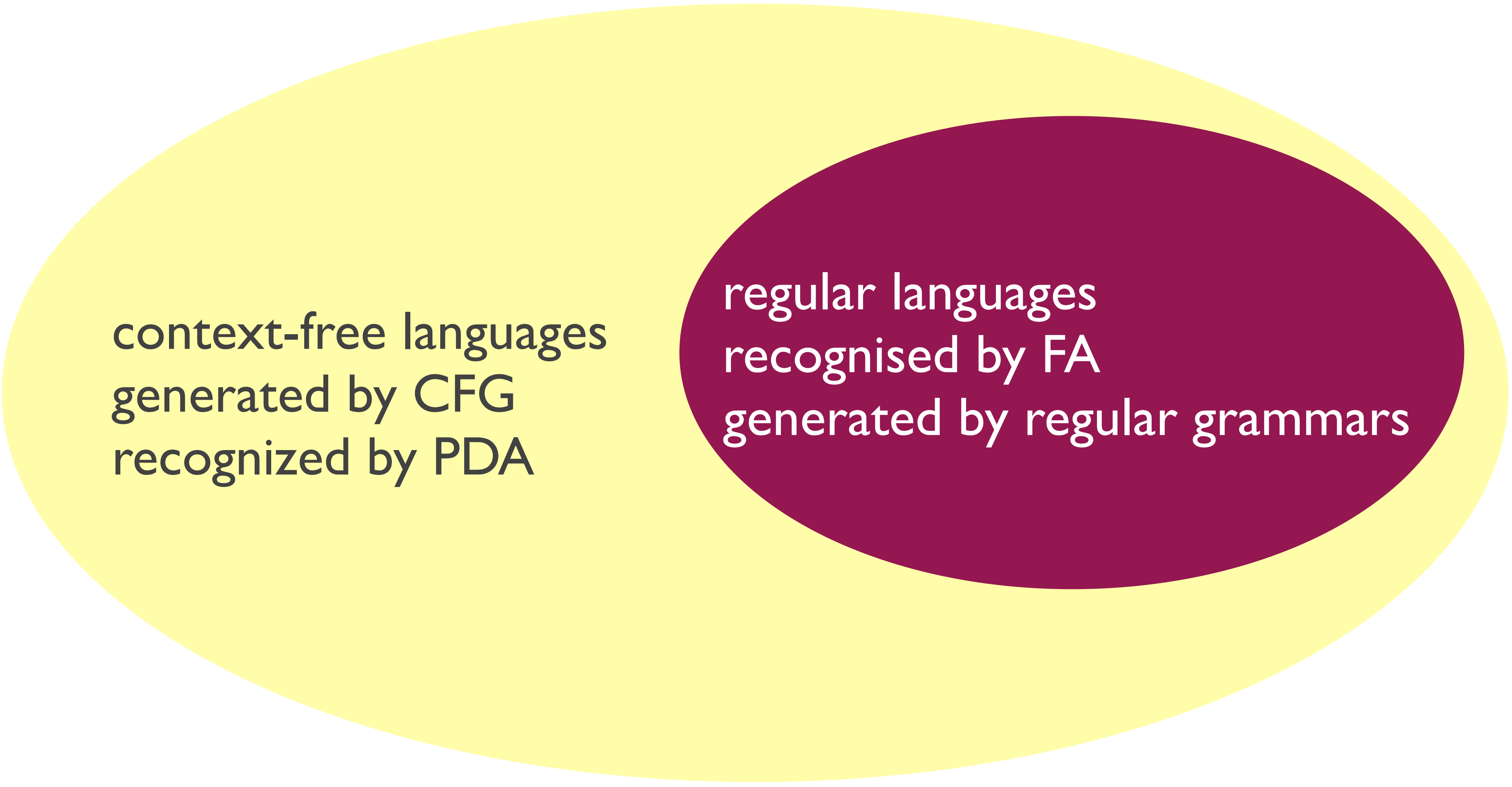

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Theorem CF4

The intersection of a regular language and a context-free language is context-free