Formale Systeme Proseminar

Tasks for Week 9, 1.12.2016

Task 1 Show with derivations that the following formula is a tautology

$$\exists_x \forall_y [P(x) \Rightarrow Q(y)] \Rightarrow (\forall_u [P(u)] \Rightarrow \exists_v [Q(v)])$$

Task 2 Prove with a derivation that the following formula is a tautology.

$$\exists_y [\forall_x [P(x) \wedge Q(x, y)]] \Rightarrow \forall_z [P(z)]$$

Task 3 Prove with a derivation that the following formula is a tautology.

$$\forall_x [P(x) : Q(x)] \Rightarrow (\exists_x [P(x)] \Rightarrow \exists_x [Q(x)])$$

Also prove it with a calculation.

Task 4 Prove with a derivation that the following formula is a tautology.

$$\exists_x [\forall_y [P(x, y)]] \Rightarrow \forall_v [\exists_u [P(u, v)]]$$

Task 5 Let $M = \{a, b, c\}$. Give $M \times M$. Define (if possible) a relation R on M that is reflexive and symmetric, but not transitive.

Task 6 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is reflexive and transitive, but not symmetric.

Task 7 Let $M = \{a, b, c\}$. Define (if possible) a relation R on M that is symmetric and transitive, but not reflexive.

Task 8 Check if the following relation is reflexive, symmetric, and/or transitive:

$$R_1 = \{(x, y) \mid x, y \in \mathbb{R},\, x = 0 \wedge y \geq 0\}.$$

Task 9 Is it possible that a relation R is both

(a) symmetric and asymmetric?

(b) symmetric and antisymmetric?