Formale Systeme Proseminar

Tasks for Week 13, 12.1.2017

Task 1 Prove Proposition S3 from the lectures, that is, show that if $f\colon A \to B$ is a surjective function and $B' \subseteq B$ then $f(f^{-1}(B')) = B'$.

Task 2 Show that the function $f\colon \mathbb{N} \to \mathbb{N}$ given by $f(n) = n + 5$ is an injection.

Task 3 Show that the function $f\colon \mathbb{Z} \to \mathbb{N}$ given by

$$f(k) = |k| = \begin{cases} k & \text{if } k \geq 0 \\ -k & \text{if } k < 0 \end{cases}$$

is a surjection.

Task 4 Let X be any set. Show that the identity function $\mathrm{id}_X\colon X \to X$ defined by $\mathrm{id}_X(x) = x$ is a bijection.

Task 5 Let $f\colon A \to B$ and $g\colon B \to C$ be two injective functions. Prove that then $g \circ f$ is injective as well. (Hence, you need to prove Lemma I4 from the lectures.)

Task 6 Prove that $f\colon A \to B$ is surjective if and only if it is right-cancelative: given any two functions $g\colon B \to C$ and $h\colon B \to C$ if $g \circ f = h \circ f$, then $g = h$.

Task 7 The sequence $(a_i \mid i \in \mathbb{N})$ is inductively defined by

$$a_0 = 2$$
$$a_{i+1} = 2a_i - 1$$

Prove (by induction) that $\forall n \in \mathbb{N}. a_n = 2^n + 1$.